VOLUME II
FLYING QUALITIES PHASE

CHAPTER 8
DYNAMICS

SEPTEMBER 1988

USAF TEST PILOT SCHOOL
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8.1 INTRODUCTION

Dynamics is concerned with the time history of the motion of physical systems. An aircraft is such a system, and its dynamic stability behavior can be predicted through mathematical analysis of the aircraft's equations of motion and verified through flight test.

In the good old days when aircraft were simple, all aircraft exhibited five characteristic dynamic modes of motion, two longitudinal and three lateral-directional modes. The two longitudinal modes are the short period and the phugoid; the three lateral-directional modes are the Dutch roll, spiral, and roll modes.

As aircraft control systems increase in complexity, it is conceivable that one or more of these modes may not exist as a dominant longitudinal or lateral-directional mode. Frequently the higher order effects of complex control systems will quickly die out and leave the basic five dynamic modes of motion. When this is not the case, the development of special procedures may be required to meaningfully describe an aircraft's dynamic motion. For the purposes of this chapter, aircraft will be assumed to possess the five basic modes of motion.

During this study of aircraft dynamics, the solutions to both first order and second order systems will be of interest, and several important descriptive parameters will be used to define the dynamic response of either a first or a second order system.

The quantification of handling qualities, that is, specifying how the magnitude of some of these descriptive parameters can be used to indicate how well an aircraft can be flown, has been an extensive investigation which is by no means complete. Flight tests, simulators, variable stability aircraft, engineering know-how, and pilot opinion surveys have all played major roles in this investigation. The military specification on aircraft handling qualities, MIL-F-8785C, is the current state-of-the-art and ensures that an aircraft will handle well if compliance has been achieved. No attempt will be made to evaluate how satisfactory MIL-F-8785C is for this purpose, but development of the skills necessary to accomplish an analysis of the dynamic behavior of an aircraft will be studied.

8.2 STATIC VS DYNAMIC STABILITY

The static stability of a physical system is concerned with the initial reaction of the system when displaced from an equilibrium condition. The system could exhibit either:

Positive static stability - initial tendency to return
Static instability - initial tendency to diverge
Neutral static stability - remain in displaced position

A physical system's dynamic stability analysis is concerned with the resulting time history motion of the system when displaced from an equilibrium condition.

8.2.1 Dynamically Stable Motions

A particular mode of an aircraft's motion is defined to be "dynamically stable" if the parameters of interest tend toward finite values as time increases without limit. Some examples of dynamically stable time histories and some terms used to describe them are shown in Figures 8.1 and 8.2.

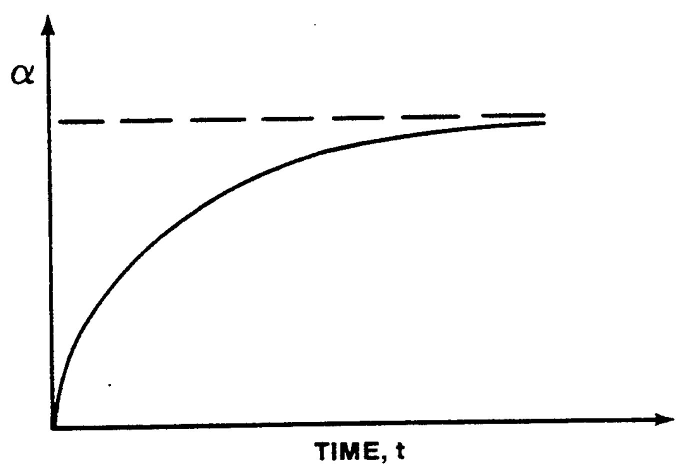


FIGURE 8.1 EXPONENTIALLY DECREASING

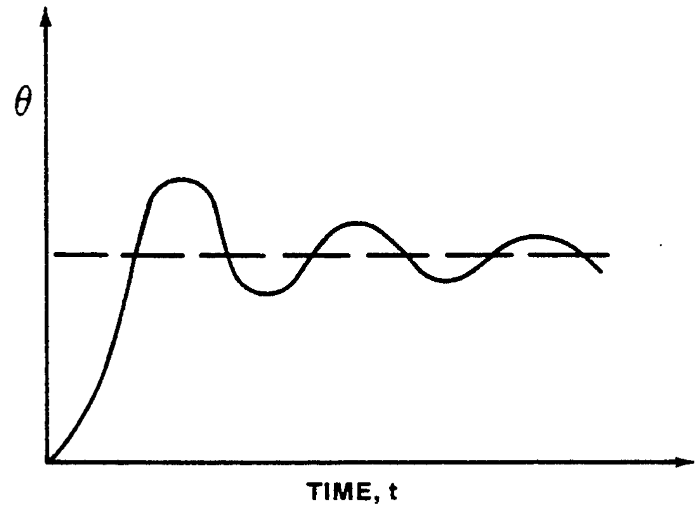


FIGURE 8.2 DAMPED SINUSOIDAL OSCILLATION

8.2.2 Dynamically Unstable Motion

A mode of motion is defined to be "dynamically unstable" if the parameters of interest increase without limit as time increases without limit. Some examples of dynamic instability are shown in Figures 8.3 and 8.4.

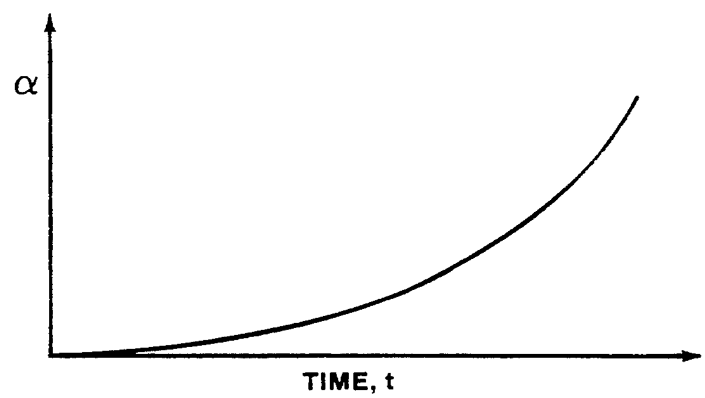


FIGURE 8.3 EXPONENTIALLY INCREASING

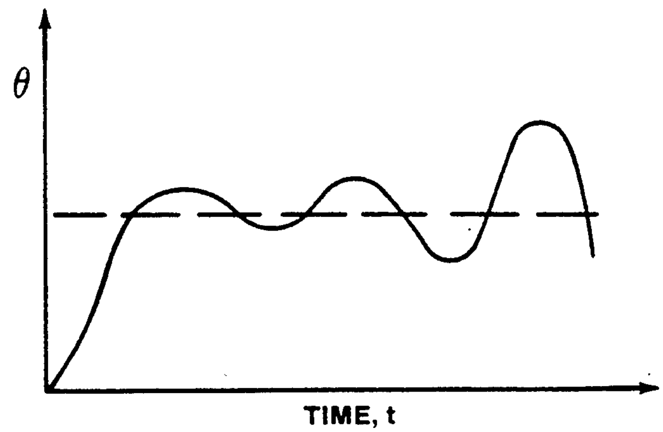


FIGURE 8.4 DIVERGENT SINUSOIDAL OSCILLATION

8.2.3 Dynamically Neutral Motion

A mode of motion is said to have "neutral dynamic stability" if the parameters of interest exhibit an undamped sinusoidal oscillation as time increases without limit. A sketch of such motion is shown in Figure 8.5.

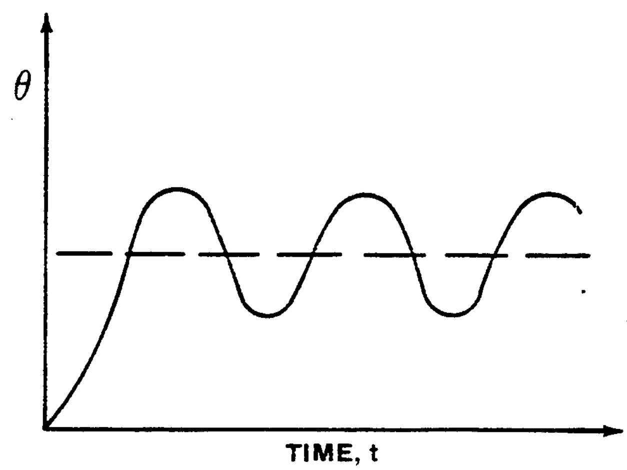
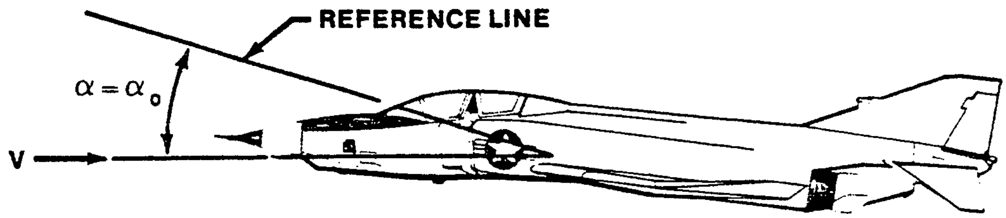


FIGURE 8.5 UNDAMPED OSCILLATION

Example Stability Problem:

A stability analysis can be accomplished to analyze the aircraft shown in Figure 8.6 for longitudinal static stability and dynamic stability. This aircraft is operating at a constant trimmed angle of attack, α_o, in 1g flight.



(SPECIFIED THAT $C_{M_\alpha} < 0$)

FIGURE 8.6 EXAMPLE STABILITY ANALYSIS

Static Stability Analysis. If the aircraft was displaced from its equilibrium flight conditions by increasing the angle of attack to $\alpha = \alpha_o + \Delta\alpha$ then the change in pitching moment due to the increase in angle of attack would be nose down because $C_{M_\alpha} < 0$. Thus, the aircraft has positive static longitudinal stability in that its initial tendency is to return to equilibrium.

Dynamic Stability Analysis. The motion of the aircraft as a function of time must be known to describe its dynamic stability. Two methods could be used to find the time history of the motion of the aircraft:

1. Solutions to the aircraft equations of motion could be obtained and analyzed.

2. A flight test could be flown in which the aircraft is perturbed from its equilibrium condition and the resulting motion is recorded and observed.

A sophisticated solution to the aircraft equations of motion with valid aerodynamic inputs can result in good theoretically obtained time histories. However, the fact remains that the only way to discover the aircraft's actual dynamic motion is to flight test and record its motion for analysis.

8.3 EXAMPLES OF FIRST AND SECOND ORDER DYNAMIC SYSTEMS

8.3.1 Second Order System with Positive Damping

The problem of finding the motion of the block shown in Figure 8.7 encompasses many of the methods and ideas that will be used in finding the time history of an aircraft's motion from its equations of motion.

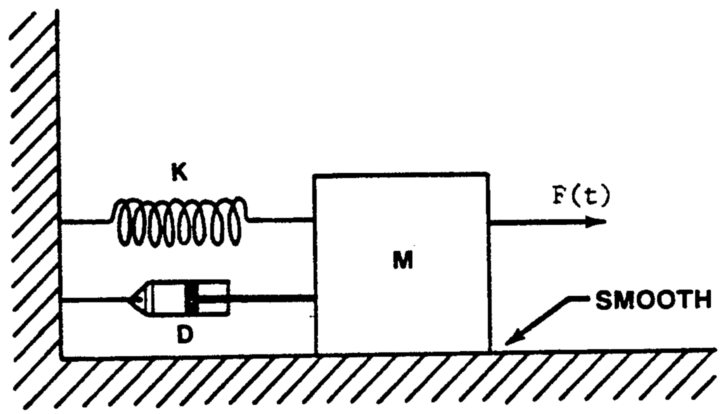


FIGURE 8.7 SECOND ORDER SYSTEM

The differential equation of motion for this physical system is

$$M\ddot{x} + D\dot{x} + Kx = F(t)$$

After Laplace transforming, assuming that the initial conditions are zero, and solving for X(s)/F(s), the transfer function, the result is

$$\frac{X(s)}{F(s)} = \frac{1/M}{s^2 + \frac{D}{M}s + \frac{K}{M}}$$

The denominator of the transfer function which gives the free response of a system will be referred to as its "characteristic equation," and the symbol $\Delta(s)$ will be used to indicate the characteristic equation.

The characteristic equation, $\Delta(s)$, of a second order system will frequently be written in a standard notation.

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \tag{8.1}$$

where

$$\omega_n = \text{natural frequency}$$

$$\zeta = \text{damping ratio.}$$

The two terms, natural frequency and damping ratio, are frequently used to characterize the motion of second order systems.

Also, knowing the location of the roots of $\Delta(s)$ on the complex plane makes it possible to immediately specify and sketch the dynamic motion associated with a system. Continuing to discuss the problem shown in Figure 8.7 and making an identity between the denominator of the transfer function and the characteristic equation

$$\omega_n = \sqrt{\frac{K}{M}}$$

$$\zeta = \frac{D}{2M\sqrt{\frac{K}{M}}}$$

The roots of $\Delta(s)$ can be found by applying the quadratic formula to the characteristic equation

$$s_{1,2} = -\zeta\omega_n \pm j\omega_d$$

Where

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

Note that if $(-1 < \zeta < 1)$, then the roots of $\Delta(s)$ comprise a complex conjugate pair, and for positive ζ would result in root locations as shown in Figure 8.8.

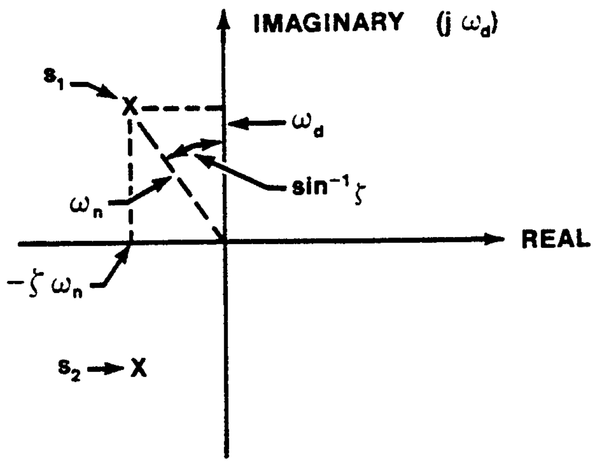


FIGURE 8.8A COMPLEX PLANE

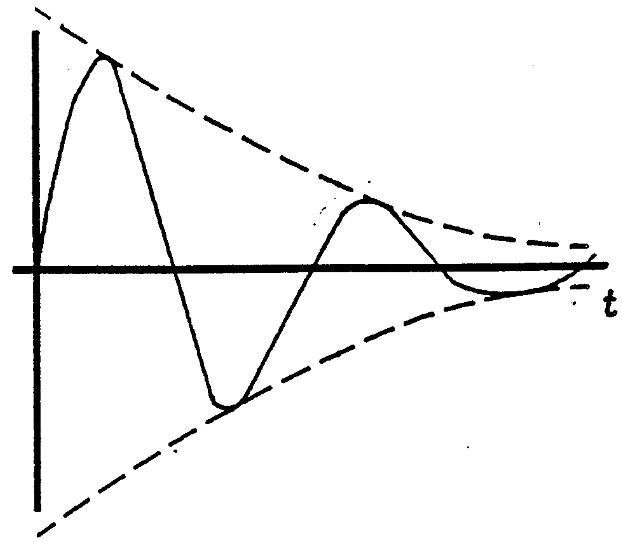


FIGURE 8.8B RESPONSE

The equation describing the time history of the block's motion can be written by knowing the roots of $\Delta(s)$, $s_{1,2}$, shown above.

$$x(t) = C_1 e^{-\zeta\omega_n t} \cos(\omega_d t + \phi)$$

Where

C_1 and ϕ are constants determined from initial conditions. Knowing either the $\Delta(s)$ root location shown in Figure 8.8A or the equation in x(t) makes it possible to sketch or describe the time history of the motion of the
block. The motion of the block shown in Figure 8.7 as a function of time is
a sinusoidal oscillation within an exponentially decaying envelope and is dynamically stable as shown in Figure 8.8B.

8.3.2 Second Order System With Negative Damping

A similar procedure to that used in the previous section can be used to find the motion of the block shown in Figure 8.9.

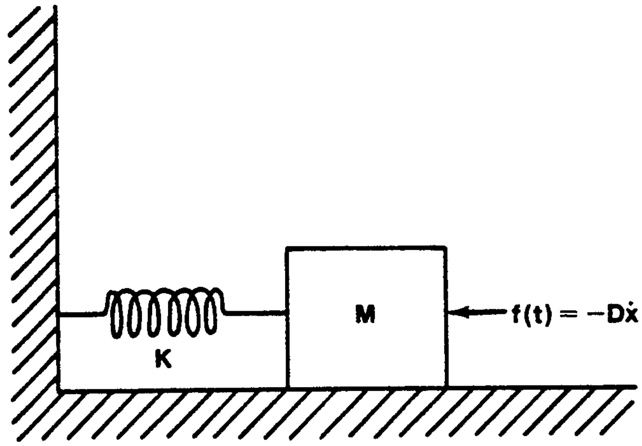


FIGURE 8.9

The differential equation of motion for this block is

$$M\ddot{x} - D\dot{x} + Kx = f(t)$$

after assuming the inital conditions are zero, the transfer function is

$$\frac{X(s)}{F(S)} = \frac{1/M}{s^2 - \frac{D}{M}s + \frac{K}{M}}$$

By inspection, for this system

$$\omega_n = \sqrt{\frac{K}{M}}$$

$$\zeta = \frac{-D}{2M\sqrt{\frac{K}{M}}}$$

Note that the damping ratio has a negative value. The equation giving the time response of this system is

$$x(t) = C_1 e^{(\text{pos. value})\, t} \cos(\omega_d t + \phi)$$

where

$$-\zeta\omega_n = \text{positive value} : \tau = \frac{1}{\zeta\omega_n}$$

For the range $(-1 < \zeta < 0)$, the roots of $\Delta(s)$ for this system could again be plotted on the complex plane from $s_{1,2} = -\zeta\omega_n \pm j\omega_d$ as shown in Figure 8.10.

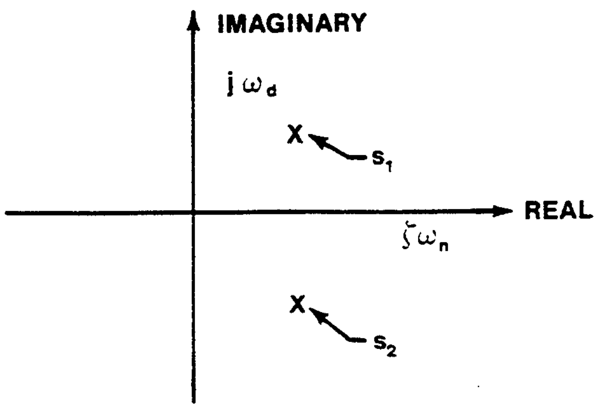


FIGURE 8.10A COMPLEX PLANE

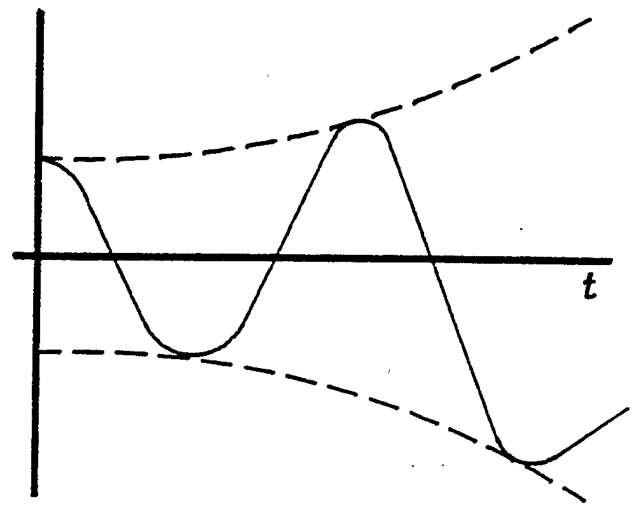


FIGURE 8.10B RESPONSE

The motion of this system can now be described. The motion of this system is a sinusoidal oscillation with an exponentially diverging envelope and is dynamically unstable as shown in Figure 8.10B.

8.3.3 Unstable First Order System:

Assume that some physical system has been mathematically modeled and its equation of motion in the s domain is

$$X(s) = \frac{.5}{.4s - .7} = \frac{1.25}{s - 1.75}$$

For this system the characteristic equation is

$$\Delta(s) = s - 1.75$$

And its root is shown plotted on the complex plane in Figure 8.11A.

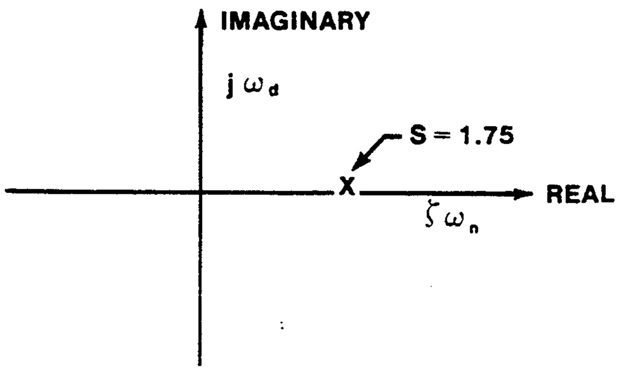


FIGURE 8.11A COMPLEX PLANE

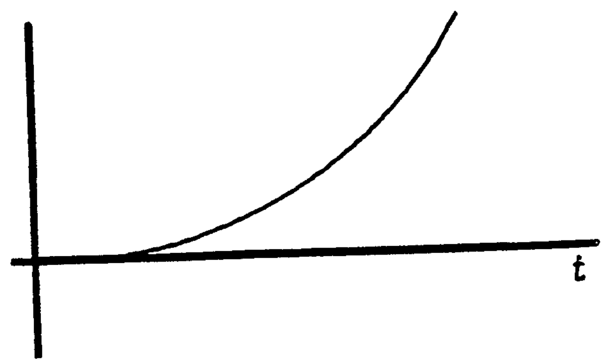


FIGURE 8.11B RESPONSE

The equation of motion in the time domain becomes

$$\phi(t) = 1.25\ e^{1.75\ t}$$

Note that it is possible to sketch or describe the motion for this system by knowing the location of the root of $\Delta(s)$ or its equation of motion.

For an unstable first order system such as this, one parameter that can be used to characterize its motion is T_2, defined as the time to double amplitude. Without proof,

$$T_2 = \frac{.693}{a} \ : \ a = -\zeta\omega_n \qquad (8.2)$$

For a first order system described by

$$z = C_1\ e^{at} = C\ e^{-\zeta\omega_n t}$$

Note that for a stable first order system, a similar parameter $T_{1/2}$ is the time to half amplitude.

$$T_{1/2} = \frac{.693}{a} = \frac{.693}{\zeta\omega_n} \tag{8.3}$$

Where the term, a, must have a negative value for a stable system.

8.3.4 Additional Terms Used To Characterize Dynamic Motion

The time constant, τ, is defined for a stable first order system as the time when the exponent of e in the system equation is -1, or time to reach 63% of final steady state value. For $C\,e^{-\zeta\omega_n t}$

$$\tau = \frac{1}{|a|} = \frac{1}{|\zeta\omega_n|} \tag{8.4}$$

The time constant can be thought of as the time required for the parameter of interest to accomplish (1 - 1/e)th of its final steady state. Note that

$$A = 1-e^{-t/\tau}$$

so for $t = \tau$

A = .63 or 63% of Final Steady State Value.

For $t = 2\tau$

A = .86 or 86% of Final Steady State Value.

Thus the magnitude of the time constant gives a measure of how quickly the dynamic motion of a first order system occurs. Small τ implies a system that, once displaced, returns to equilibrium quickly.

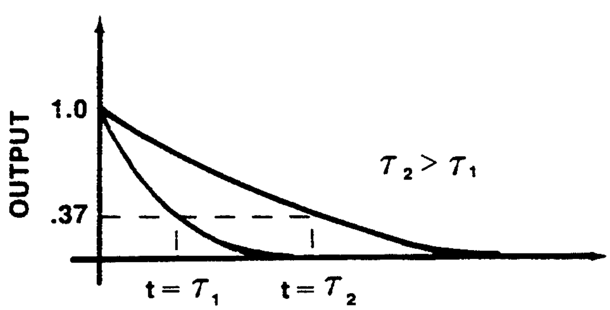


FIGURE 8.12 FIRST ORDER TIME RESPONSE

For a second order system we measure how quickly the envelope of oscillation changes.

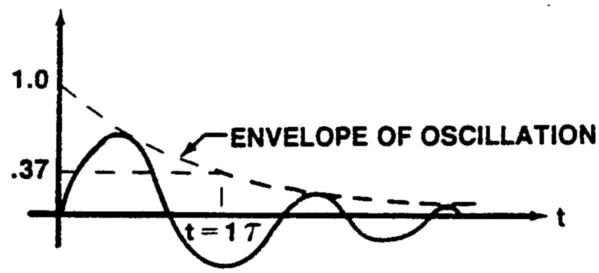


FIGURE 8.13 SECOND ORDER TIME RESPONSE

Final or steady state value for a given set of equations can be determined using the following expression

$$\text{Final Steady State Value} = \lim_{s \to 0} [s\, F(s)]$$

where F(s) is the Laplace transform of the set of equations with initial conditions equal to zero.

If the Laplace transform of f(t) is F(s), and if $\lim_{t \to \infty} f(t)$ exists, then

$$\lim_{s \to 0} sF(s) = \lim_{t \to \infty} f(t)$$

Example: $f(t) = A\ddot{\theta}(t) + B\dot{\theta}(t) + C\theta(t)$

Input function $= D\, \delta_e(t)$

where $\delta_e(t)$ is a unit step function.

Taking the Laplace transform with the initial conditions equal to zero.

$$F(s) = \theta(s) = \frac{\frac{D}{A}}{s^2 + \frac{B}{A}s + \frac{C}{A}}\, \delta_e(s)$$

Note: $\mathcal{L}$ [Step function] = 1/s

$\mathcal{L}$ [Impulse Function] = 1

$$\theta_{ss} = \lim_{s \to 0} [s\, F(s)] = \lim_{s \to 0} s \left[\frac{\frac{D}{A}}{s^2 + \frac{B}{A}s + \frac{C}{A}}\right]\left(\frac{1}{s}\right)$$

$$\theta_{ss} = \lim_{s \to 0} s \left[\frac{\frac{D}{A}}{s\left(s^2 + \frac{B}{A} s + \frac{C}{A}\right)} \right]$$

The final steady state value of θ due to a unit step function input is

$$\theta_{ss} = D/C$$

The following list contains some terms commonly used to describe second order system response based on damping ratio values:

Terms	Damping Ratio Value
Overdamped	$1 < \zeta$
Critically damped	$1 = \zeta$
Underdamped	$0 < \zeta < 1$
Undamped	$0 = \zeta$
Negatively damped	$\zeta < 0$

SUMMARY OF DYNAMIC RESPONSE PARAMETERS

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s_{1,2} = -\zeta\omega_n \pm j\omega_d$$

ζ – Damping Ratio

ω_n – Undamped Natural Frequency

ω_d Damped Frequency

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

τ – Time Constant

$$\tau = \frac{1}{\zeta\omega_n}$$

T – Period

$$T = \frac{2\pi}{\omega_d}$$

$t_{1/2}$ Time to Half Amplitude

$$t_{1/2} = \frac{.69}{\zeta\omega_n}$$

8.4 THE COMPLEX PLANE

It is possible to describe the type response a system will have by knowing the location of the roots of its characteristic equation on the complex plane. A first order response will be associated with each real root, and a complex conjugate pair will have a second order response that is either stable, neutrally stable, or unstable. A complicated system such as an aircraft might have a characteristic equation with several roots, and the total response of such a system will be the sum of the responses associated with each root. A summary of root location and associated response is presented in the following list and in Figures 8.12B and 8.12C.

	Root Location	Associated Response
Case I	On the negative Real axis (1st. Order Response)	Dynamically stable with exponential decay
Case II	In the left half plane off the negative Real axis (2nd. Order Response)	Dynamically stable with sinusoidal oscillation in exponentially decaying envelope
Case III	On the Imaginary axis (2nd. Order Response)	Neutral dynamic stability
Case IV	In the right half plane off the positive Real axis (2nd. Order Response)	Dynamically unstable with sinusoidal oscillation in exponentially increasing envelope
Case V	On the positive Real axis (1st. Order Response)	Dynamically unstable with exponential increase

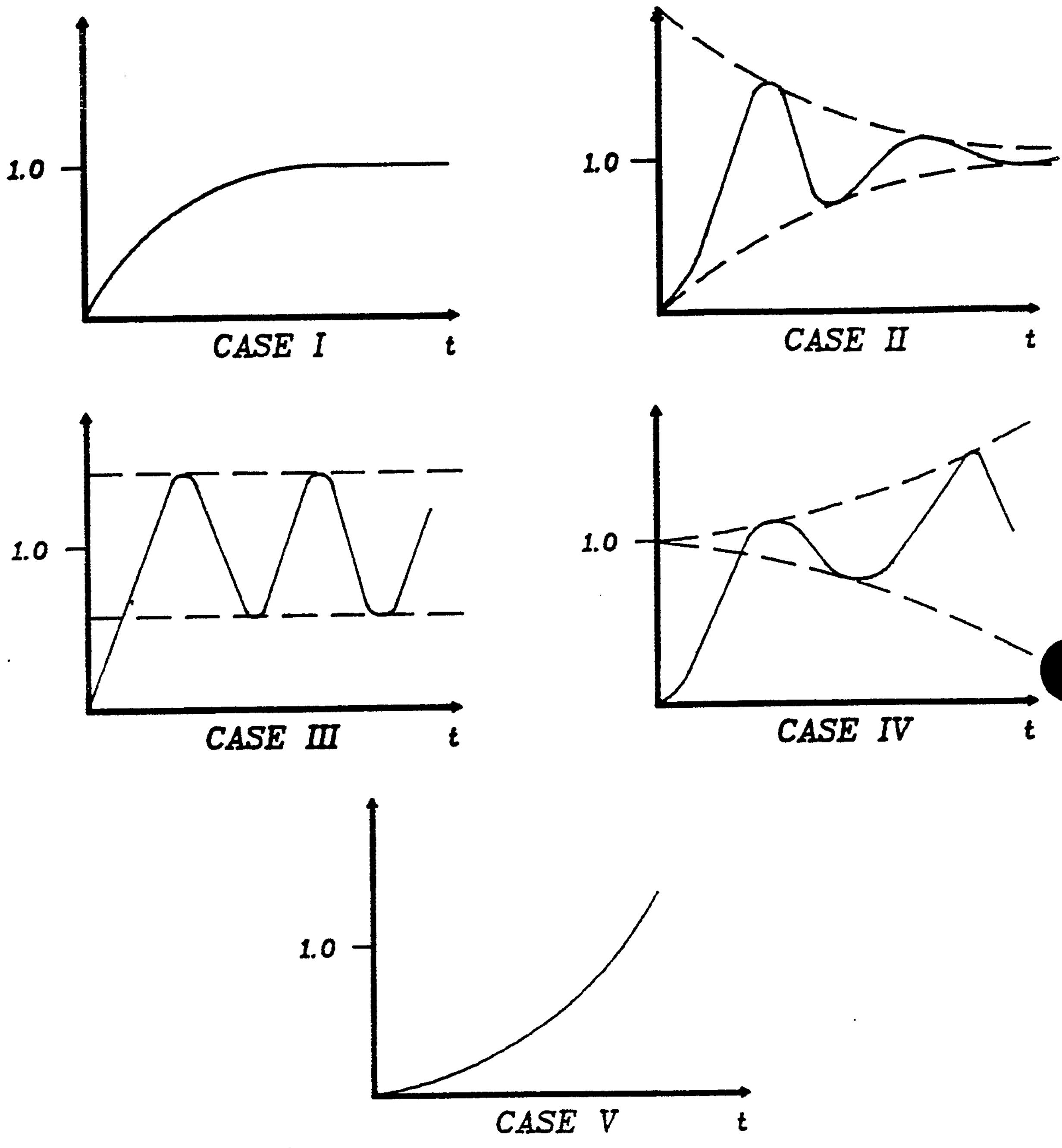


FIGURE 8.14 POSSIBLE 2ND. ORDER ROOT RESPONSES TO A UNIT STEP INPUT

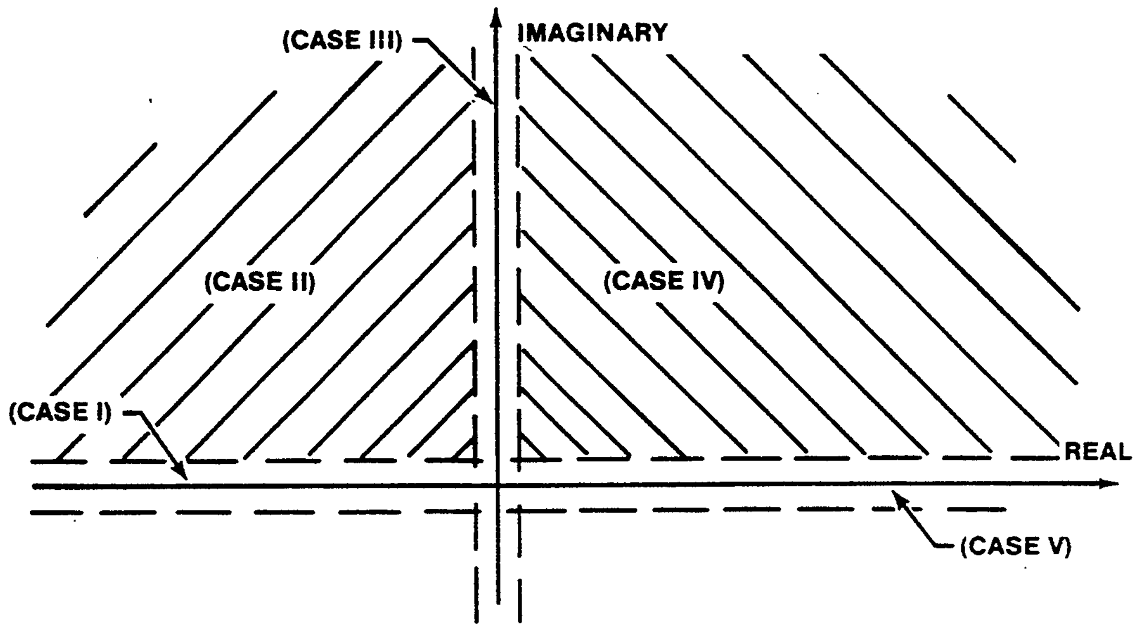


FIGURE 8.15 ROOT LOCATION IN THE COMPLEX PLANE

8.5 EQUATIONS OF MOTION

Six equations of motion (three translational and three rotational) for a rigid body flight vehicle are required to solve its motion problem. If a rigid body aircraft and constant mass are assumed, then the equations of motion can be derived and expressed in terms of a coordinate system fixed in the body. Solving for the motion of a rigid body in terms of a body fixed coordinate system is particularly convenient in the case of an aircraft when the applied forces are most easily specified in the body axis system.

"Stability axes" can be used as the specified coordinate system. With the vehicle at reference flight conditions, the x axis is aligned into the relative wind; the z axis is 90° from the x axis in the aircraft plane of symmetry, with positive direction down relative to the vehicle; and the y axis completes the orthogonal triad. This xyz coordinate system is then fixed in the vehicle and rotates with it when perturbed from the reference equilibrium conditions. The solid lines in Figure 8.16 depict initial alignment of the stability axes, and the dashed lines show the perturbed coordinate system.

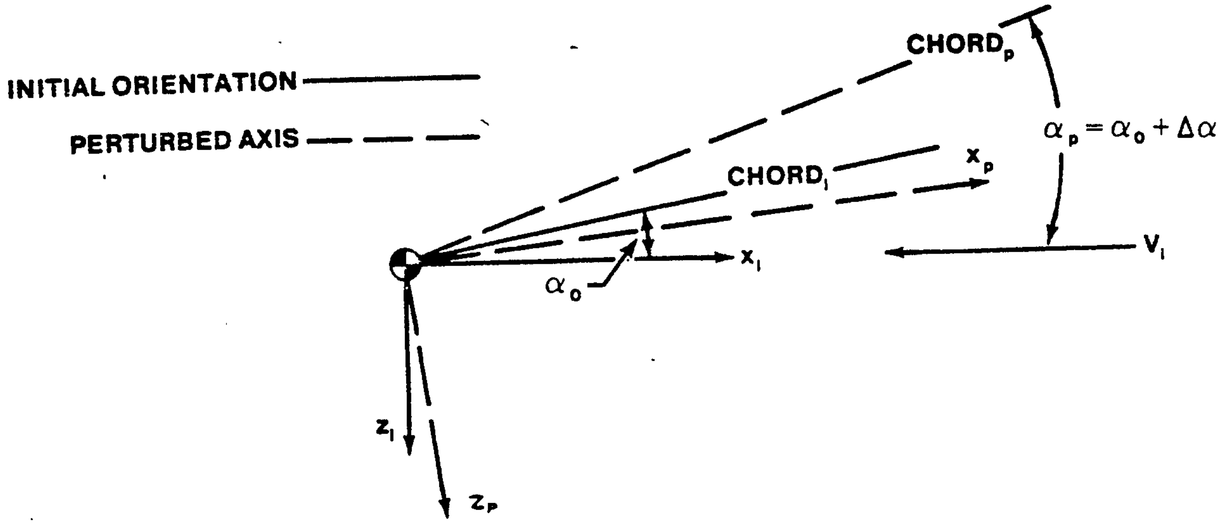


FIGURE 8.16 STABILITY AXIS SYSTEM

Chapter 4, Equations of Motion, Pg. 4.52, contains the derivation of the complete equations of motion, and the results are listed here:

$$
\begin{aligned}
F_x &= m(\dot{u} + qw - rv) \\
F_y &= m(\dot{v} + rU - pw) \\
F_z &= m(\dot{w} + pv - qU) \\
\mathcal{L} &= \dot{p}I_x + qr(I_z - I_y) - (\dot{r} + pq)I_{xz} \\
\mathcal{M} &= \dot{q}I_y - pr(I_z - I_x) + (p^2 - r^2)I_{xz} \\
\mathcal{N} &= \dot{r}I_z + pq(I_y - I_x) + (qr - \dot{p})I_z
\end{aligned}
\tag{8.5}
$$

where F_x, F_y, and F_z are forces in the x, y, and z direction, and $\mathcal{L}$, $\mathcal{M}$, and $\mathcal{N}$ are moments about the x, y, and z axes taken at the vehicle center of gravity and $U = U_o + u$.

Separation of the Equations of Motion:

When all lateral-directional forces, moments, and accelerations are constrained to be zero, the equations which govern pure longitudinal motion result from the six general equations of motion. That is, substituting

$$
\begin{aligned}
p &= 0 = r \\
\dot{p} &= 0 = \dot{r} \\
\mathcal{L} &= 0 = \mathcal{N} \\
F_y &= 0 \\
v &= 0 \\
\dot{v} &= 0
\end{aligned}
\tag{8.6}
$$

into the Equations labeled 8.5 results in the longitudinal equations of motion

$$
\begin{aligned}
F_x &= m\,(\dot{u} + qw) \\
F_z &= m\,(\dot{w} - qU) \\
\mathcal{M} &= \dot{q}\, I_y
\end{aligned}
\tag{8.7}
$$

Performing a Taylor series expansion of Equations labeled 8.7 as a function of u, q, and w and assuming small perturbations ($U = U_o + u$) results in a linearized set of equations for longitudinal motion. Note that the resulting equations are the longitudinal perturbation equations and that the unknowns are the perturbed values of α, u, and θ from an equilibrium condition. These equations in coefficient form are

$$
\begin{aligned}
&\frac{2m}{\rho S U_o}\dot{\hat{u}} + 2C_{D_o}\hat{u} + C_{D_u}\hat{u} + C_{D_\alpha}\alpha + C_{L_o}\theta = C_{D_{\delta_e}}\delta_e \\
&- C_{L_u}\hat{u} - 2C_{L_o}\hat{u} - \frac{2m}{\rho S U_o}\alpha - C_{L_\alpha}\alpha - \frac{c}{2U_o}C_{L_{\dot{\alpha}}}\dot{\alpha} + \frac{2m}{\rho S U_o}\dot{\theta} \\
&- \frac{c}{2U_o}C_{L_q}\dot{\theta} = C_{L_{\delta_e}}\delta_e \\
&- C_{m_u}\hat{u} - \frac{c}{2U_o}C_{m_{\dot{\alpha}}}\dot{\alpha} - C_{m_\alpha}\alpha + \frac{2I_y}{\rho S U_o^{\,2} c}\ddot{\theta} - \frac{c}{2U_o}C_{m_q}\dot{\theta} = C_{m_{\delta_e}}\delta_e
\end{aligned}
\tag{8.8}
$$

Where:

$\hat{u} = \frac{u}{U_o}$ (A dimensionless velocity parameter has been defined for convenience.)

α, θ are perturbations about their equilibrium values.

C_{D_u}, C_{L_α}, etc., are partial derivatives evaluated at the reference conditions with respect to force coefficients.

Note that these equations are for pure longitudinal motion.

Solutions to the longitudinal perturbation equations can be obtained using Laplace transforms. Taking Laplace transforms of the pitching moment equation and stating that initial perturbation values are zero results in

$$- C_{m_u} \hat{u}(s) + \left[\frac{2I_y}{\rho U_o^2 Sc} s^2 - \frac{c}{2U_o} C_{m_q} s\right]\theta(s) - \left[\frac{c}{2U_o} C_{m_{\dot{\alpha}}} s + C_{m_\alpha}\right]\alpha(s) \tag{8.9}$$

$$= C_{m_{\delta_e}} \delta_e(s)$$

The other two equations could similarly be Laplace transformed to obtain a set of longitudinal perturbation equations in the s domain.

8.5.1 <u>Longitudinal Motion</u>

The Equations 8.8 describe the longitudinal motion of an aircraft about some equilibrium conditions. The theoretical solutions for aircraft motion can be quite good, depending on the accuracy of the various aerodynamic parameters. For example, C_{D_α} is one parameter appearing in the drag force equation, and the goodness of the solution will depend on how accurately the value of C_{D_α} is known. Before an aircraft flies, such values for the various stability derivatives can be extracted from wind tunnel data.

8.5.1.1 <u>Longitudinal Modes of Motion</u> Experience has shown that aircraft exhibit two different types of longitudinal oscillations:

1. One of <u>short period</u> with relatively heavy damping that is called the "short period" mode (sp).

2. Another of long period with very light damping that is called the "phugoid" mode (p).

The periods and damping of these oscillations vary with aircraft configuration and with flight conditions.

The short period is characterized primarily by variations in angle of attack and pitch angle with very little change in forward speed. Relative to the phugoid, the short period has a high frequency and heavy damping.

Typical values for its damped period are in the range of two to five seconds. Generally, the short period motion is the more important longitudinal mode for handling qualities since it contributes to the motion being observed by the pilot when the pilot is in the loop.

The phugoid is characterized mainly by variations in u and θ with α nearly constant. This long period oscillation can be thought of as a constant total energy problem with exchanges between potential and kinetic energy. The aircraft nose drops and airspeed increases as the aircraft descends below its initial altitude. Then the nose rotates up, causing the aircraft to climb above its initial altitude with airspeed decreasing until the nose lazily drops below the horizon at the top of the maneuver.

Because of light damping, many cycles are required for this motion to damp out. However, its long period combined with low damping results in an oscillation that is easily controlled by the pilot, even for a slightly divergent motion. When the pilot is in the loop, he is frequently not aware that the phugoid mode exists as he makes control inputs and obtains aircraft response before the phugoid can be seen. Typical values for its damped period range in the order of 45 to 90 seconds.

Phugoid - Small ω_n
- Large time constant
- Small damping ratio

Short Period - Large ω_n
- Small time constant
- High damping ratio

Example:

Given a T-38 aircraft at M = .8, altitude = 20,000 ft., and at a gross weight of 9,000 lbs, the longitudinal equations in the Laplace domain become (ICs = 0)

$$[1.565s + .0045]\,\hat{u}(s) - .042\alpha(s) + .0605\,\theta(s) = C_{D_{\delta_e}}\,\delta_e(s)$$

$$.236\hat{u}(s) + [3.13s + 5.026]\alpha(s) - 3.15s\theta(s) = C_{L_{\delta_e}}\,\delta_e(s)$$

$$0 + .16\alpha(s) + [.0489s^2 - .039s]\theta(s) = C_{m_{\delta_e}}\,\delta_e(s)$$

These equations are of the form

$$a\hat{u} + b\alpha + c\theta = d\delta_e$$

$$e\hat{u} + f\alpha + g\theta = h\delta_e$$

$$i\hat{u} + j\alpha + k\theta = l\delta_e$$

and using Cramers Rule, this set of equations can be readily solved for any of the variables.

$$\begin{array}{c} (\hat{u} \quad \alpha \quad \theta) \\ \dfrac{\alpha(s)}{\delta_e(s)} = \dfrac{\begin{vmatrix} a & d & c \\ e & h & g \\ i & l & k \end{vmatrix}}{\begin{vmatrix} a & b & c \\ e & f & g \\ i & j & k \end{vmatrix}} = \dfrac{\text{Numerator}(s)}{\text{Denominator}(s)} \end{array}$$

Recall that the denominator of the above equation in the s domain is the system characteristic equation and that the location of the roots of $\Delta(s)$ will

indicate the type of dynamic response. Solving for the determinant of the denominator yields:

$$\Delta(s) \;=\; .239s^4 + .577s^3 + 1.0996s^2 + .00355s + .0028 \;=\; 0$$

Factoring this equation into two quadratics

$$\Delta(s) \;=\; (s^2 + .00216s + .00208)\,(s^2 + 2.408s + 4.595) \;=\; 0$$

standard format

$$\left(s^2 + 2\zeta\omega_{n_p} s + \omega_{n_p}^2\right)\left(s^2 + 2\zeta\omega_{n_{sp}} s + \omega_{n_{sp}}^2\right) \tag{8.10}$$

Each of the quadratics listed in the equation prior to Equation 8.10 will have a natural frequency and damping ratio associated with it, and the values can be computed by comparing the particular quadratic to the standard notation second order characteristic Equation 8.10.

LONGITUDINAL MODES OF MOTION
T-38 EXAMPLE

	Phugoid	Short Period
ζ	.0236	.562
ω_n	.0456 rad/sec	2.143 rad/sec
τ	925.9 sec	.83 sec
T	137.5 sec	3.54 sec

Roots of $\Delta(s)$ for longitudinal motion:

$$\text{phugoid roots:} \quad s_{1,2} = -.00108 \pm j\ .0456$$

$$\text{short period roots:} \quad s_{3,4} = -1.204 \pm j\ 1.733$$

These roots can then be plotted on the s-plane:

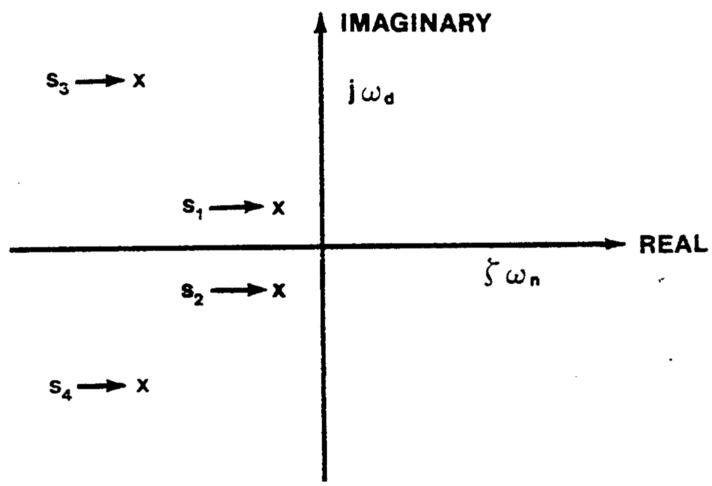


FIGURE 8.17 LONGITUDINAL MOTION COMPLEX PLANE

8.5.1.2 Short Period Mode Approximation. For a one degree of freedom first approximation, the short period is observed to be primarily a pitching motion (Figure 8.18). In addition, the short period motion occurs at nearly constant airspeed, $\Delta u = 0$; and since there is no vertical motion, changes in angle of attack are equal to changes in pitch angle, $\Delta\alpha = \Delta\theta$. With these assumptions applied to the pitching moment equation (Equation 8.8), the results become:

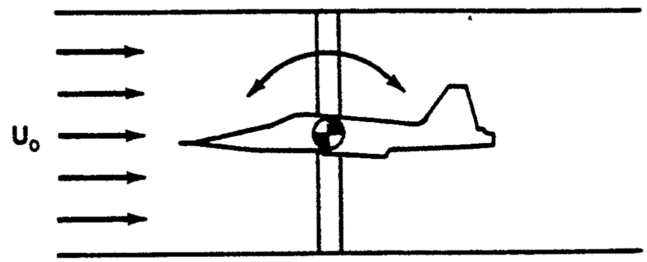


FIGURE 8.18 1 DEGREE OF FREEDOM MODEL

$$\frac{2I_Y}{\rho U_o^{\,2} Sc}\ \ddot{\alpha} - \frac{c}{2U_o} C_{m_q}\ \dot{\alpha} - C_{m_\alpha}\ \alpha = C_{m_{\delta_e}}\ \delta_e \qquad (8.11)$$

Where C_{m_u} and $C_{m_{\dot{\alpha}}}$ have been assumed to be negligible.

Applying the Laplace transform to Equation 8.11 and forming the transfer function $\alpha(s)/\delta_e(s)$ results in an approximate form of the characteristic equation.

$$\Delta(s) = \frac{I_Y}{\frac{1}{2}\ \rho U_o^{\,2} Sc}\ s^2 - \frac{c}{2U_o} C_{m_q}\ s - C_{m_\alpha} = 0 \qquad (8.12)$$

Comparison of Equation 8.12 with the standard form of the characteristic equation,(Equation 8.1), results in approximations for the short period frequency and damping ratio.

$$\zeta_{SP} = -\frac{C_{m_q}}{\sqrt{-C_{m_\alpha}}}\ \frac{c}{4U_o}\ \sqrt{\frac{\rho U_o^{\,2} Sc}{2I_Y}} \qquad (8.13)$$

$$\omega_{n_{SP}} = \sqrt{-C_{m_\alpha}\ \frac{\rho U_o^{\,2} Sc}{2I_Y}} \qquad (8.14)$$

C_{m_α} - Coefficient of pitching moment due to a change in angle of attack. Proportional to the angular displacement from equilibrium (spring constant)

C_{m_q} - Coefficient of pitching moment due to a change in pitch rate. Proportional to the angular rate (viscous damper)

Both Equations 8.13 and 8.14 can be used to predict trends expected in the short period damping ratio and natural frequency as flight conditions and aircraft configurations change. In addition, these equations show the predominant stability derivatives which affect the short period damping ratio and natural frequency.

8.5.1.3 Equation for Ratio of Load Factor to Angle of Attack Change. The requirements of MIL-F-8785C for the short period natural frequency are stated as a function of n/α and ω_n.

An expression for the slope of the lift curve is

$$C_{L_\alpha} = \frac{\Delta C_L}{\Delta \alpha} \quad \text{and} \quad \Delta C_L = \frac{\Delta n W}{\frac{1}{2} \rho U_o^{\,2} S} \tag{8.15}$$

$$\frac{\Delta n}{\Delta \alpha} = \frac{1/2 \ \rho \ U_o^{\,2} \ S \ C_{L_\alpha}}{W} \tag{8.16}$$

8.5.1.4 Phugoid Mode Approximation Equations. An approach similar to that used when obtaining the short period approximation will be used to obtain a set of equations to approximate the phugoid oscillation. Recalling that the phugoid motion occurs at nearly constant angle of attack, it is logical to substitute $\alpha = 0$ into the longitudinal motion equations. This results in a set of three equations with only two unknowns. Reasoning that the phugoid motion is characterized primarily by altitude excursions and changes in aircraft

speed, implies that the lift force and drag force equations are the two equations which should be used. The resulting set of two equations for the phugoid approximation in the Laplace domain is

$$\left[\frac{2m}{\rho S U_o} s + 2C_{D_o}\right] \hat{u}(s) + C_{L_o} \theta(s) = C_{D_{\delta_e}} \delta_e(s) \tag{8.17}$$

$$-2 C_{L_o} \hat{u}(s) + \frac{2m}{\rho S U_o} s \theta(s) = C_{L_{\delta_e}} \delta_e(s) \tag{8.18}$$

Where $C_{D_{\dot{u}}}$, C_{D_α}, C_{L_u}, C_{L_α}, $C_{L_{\dot{\alpha}}}$ and C_{L_q} have been assumed to be negligibly small.

The characteristic equation for the phugoid approximation can now be found using the above equation.

$$\Delta(s) = \left[\frac{2m}{\rho S U_o}\right]^2 s^2 + \frac{4m}{\rho S U_o} C_{D_o} s + 2 C_{L_o}^{\ 2} = 0 \tag{8.19}$$

Note that lift and weight are not equal during phugoid motion, but also realize that the net difference between lift and weight is quite small. If the approximation is made that

$$L = W$$

and then the substitution that

$$W = mg$$

it can be written that

$$\frac{2mg}{\rho S U_o^{\ 2}} = C_L$$

The phugoid characteristic equation can thus be rewritten as

$$\frac{C_{L_o}^{\,2} U_o^{\,2}}{g^2} s^2 + 2 \frac{C_{L_o} U_o}{g} C_{D_o} s + 2 C_{L_o}^{\,2} = 0 \qquad (8.20)$$

$$s^2 + 2 \frac{g}{U_o} \frac{C_{D_o}}{C_{L_o}} s + \frac{2 g^2}{U_o^{\,2}} = 0 \qquad (8.21)$$

Comparison of Equation 8.21 with the standard form of Equation 8.1 results in a simplified approximate expression for phugoid natural frequency and is given by

$$\omega_{n_p} = \frac{45.5}{U_o} \qquad (8.22)$$

Where U_o is true velocity in feet per second.

A simplified approximate expression for the phugoid damping ratio can also be obtained and is given by

$$\zeta_p = \frac{1}{\sqrt{2}} \frac{C_D}{C_L} \qquad (8.23)$$

Equations 8.22 and 8.23 can be used to understand some major contributors to the natural frequency and damping ratio of the phugoid motion.

8.5.2 Lateral Directional Motion Mode

There are three typical asymmetric modes of motion exhibited by aircraft. These modes are the roll, spiral, and Dutch roll.

8.5.2.1 Roll Mode The roll mode is considered to be a first order response which describes the aircraft roll rate response to an aileron input. Figure 8.19 depicts an idealized roll rate time history to a step aileron input. Most aircraft take from one to three seconds to reach steady state roll rate.

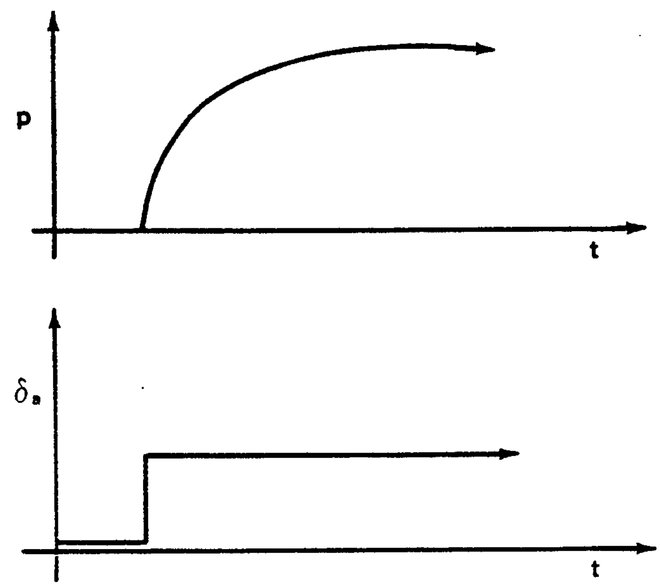


FIGURE 8.19 TYPICAL ROLL MODE

8.5.2.2 Spiral Mode The spiral mode is considered to be a first order response which describes the aircraft bank angle time history as it tends to increase or decrease from a small, nonzero bank angle. After a wings level trim shot, the spiral mode can be observed by releasing the aircraft from bank angles as great as 20° and allowing the spiral mode to occur without control inputs. If this mode is divergent, the aircraft nose continues to drop as the bank angle continues to increase, resulting in the name "spiral mode."

8.5.2.3 Dutch Roll Mode The Dutch roll mode is a coupled yawing and rolling motion lightly damped, moderately low frequency oscillation. Typically, as the aircraft nose yaws to the right, a right roll due to the yawing motion is generated. This causes increased lift and induced drag on the left wing, and the nose yaws to the left. The combination of restoring forces and moments, damping, and aircraft inertia is generally such that after the motion peaks out to the right, a nose left yawing motion begins accompanied by a roll to the left.

One of the pertinent Dutch roll parameters is ϕ/β, the ratio of bank angle to sideslip angle which may be represented by

$$\phi/\beta = f\left(\frac{C_{l_\beta}}{C_{n_\beta}}\right) \tag{8.24}$$

A very low value for ϕ/β implies little bank change during Dutch roll. In the limit when ϕ/β is zero, the Dutch roll motion consists of a pure yawing motion that most pilots consider less objectionable than the Dutch roll mode with a high value of ϕ/β.

A rudder doublet is frequently used to excite the Dutch roll; Figure 8.20 shows a typical Dutch roll time history.

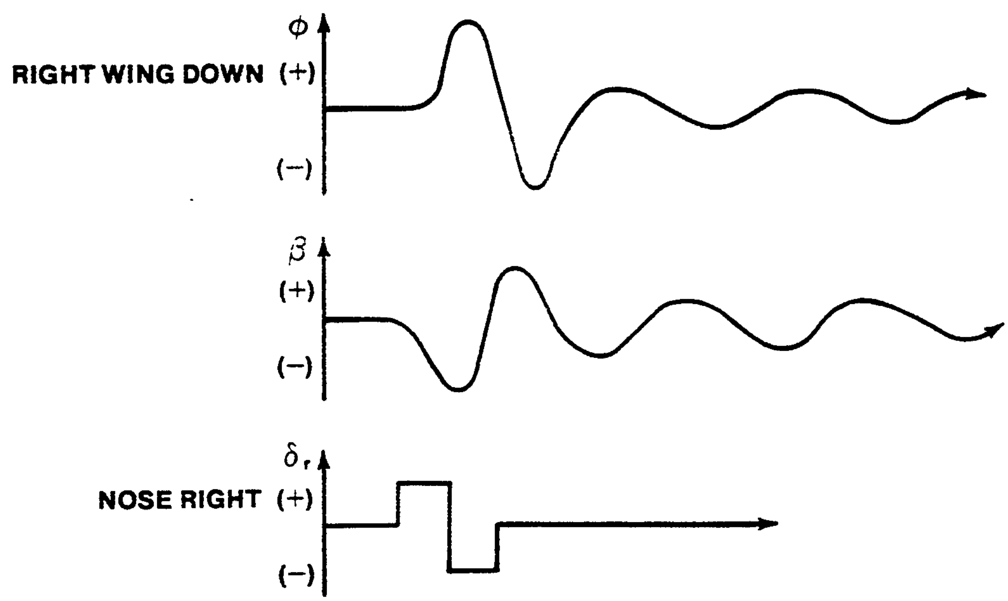


FIGURE 8.20 TYPICAL DUTCH ROLL MODE

8.5.3 Asymmetric Equations of Motion

Similar to the separation of the longitudinal equations, the set of equations which describes lateral-directional motion can be separated from the six general equations of motion. Starting with equilibrium conditions and specifying that only asymmetric forcing functions, velocities, and accelerations exist, results in the lateral directional equations of motion. Assuming small perturbations and using a linear Taylor Series approximation for the forcing functions result in the linear, lateral directional perturbation equations of motion.

Side Force:

$$-\frac{b}{2U_o} C_{y_p} \dot{\phi} - C_{L_o} \phi + \left[\frac{2m}{\rho S U_o} - \frac{b}{2U_o} C_{y_r}\right] \dot{\psi} + \frac{2m}{\rho S U_o} \dot{\beta} - C_{y_\beta} \beta = C_{y_{\delta_r}} \delta_r + C_{y_{\delta_a}} \delta_a$$

Rolling Moment:

$$\frac{2I_x}{\rho S b U_o^2} \ddot{\phi} - \frac{b}{2U_o} C_{l_p} \dot{\phi} + \frac{2I_{xz}}{\rho S b U_o^2} \ddot{\psi} - \frac{b}{2U_o} C_{l_r} \dot{\psi} - C_{l_\beta} \beta = C_{l_{\delta_r}} \delta_r + C_{l_{\delta_a}} \delta_a \quad (8.25)$$

Yawing Moment:

$$-\frac{2I_{xz}}{\rho S b U_o^2} \ddot{\phi} - \frac{b}{2U_o} C_{n_p} \dot{\phi} + \frac{2I_z}{\rho S b U_o^2} \ddot{\psi} - \frac{b}{2U_o} C_{n_r} \dot{\psi} - C_{n_\beta} \beta = C_{n_{\delta_r}} \delta_r + C_{n_{\delta_a}} \delta_a$$

The lateral-directional equations of motion have been non dimensionalized by span, b, as opposed to chord. The stability derivative C_{l_p} is not a lift referenced stability derivative but the script l refers to rolling moment. If the perturbation products of inertia are not small, then the lateral-directional motion will couple directly into longitudinal motion as seen from the pitching moment equation (Equation 8.5). Our analysis assumes that coupling does not exist.

8.5.3.1 Roots Of $\Delta(s)$ For Asymmetric Motion The roots of the lateral-directional characteristic equation typically are comprised of a relatively large negative real root, a small root that is either positive or negative, and a complex conjugate pair of roots.

The large real root is the one associated with the roll mode of motion. Note that a large negative value for this root implies a fast time constant.

The small real root that might be either positive or negative is associated with the spiral mode. A slowly changing time response results from

this small root, and the motion is either stable for a negative root or divergent for a positive root.

The complex conjugate pair of roots corresponds to the Dutch roll mode, which frequently exhibits high frequency and light damping. This second order motion is of great interest in handling qualities investigations.

For the T-38 example (Pg. 8.22), the lateral directional characteristic equation is: $(s^2 + .87\ s + 18.4)_{1,2}\ (s + 6.822)_3\ (s + .00955)_4 = 0$

Lateral-Directional Modes of Motion - T-38 Example

	SPIRAL_4	ROLL_3	$\text{DUTCH ROLL}_{1,2}$
ω_n	——	——	4.29 rad/sec
ζ	——	——	.102
τ	105 sec	.1465 sec	2.31 sec

8.5.3.2 Approximate Roll Mode Equation. This approximation results from the hypothesis that only rolling motion exists and use of the rolling moment equation results in the roll mode approximation equation.

$$\left[\frac{2I_x}{\rho S b U_o{}^2}\ s - \frac{b}{2U_o} C_{1_p}\right] \dot{\phi}(s) = C_{1_{\delta_a}}\ \delta_a \qquad (8.26)$$

The roll mode characteristic equation root is

$$s_R = \frac{b^2\ S\ \rho\ U_o}{4\ I_x}\ C_{1_p} \qquad (8.27)$$

$$\text{NOTE:}\quad \tau_R = \frac{1}{s_R} = \frac{2I_x\ U_o}{b^2 S}\ \frac{1}{\frac{1}{2}\rho U_o{}^2}\ \frac{1}{C_{1_p}}$$

Note that C_{1_p} less than zero implies stability for the roll mode and that a larger negative value of s_R implies an aircraft that approaches its steady

state roll rate quickly. A functional analysis can be made using Equation 8.27 to predict trends in τ_R, the roll mode time constant, as flight conditions change.

8.5.3.3 Spiral Mode Stability. After the Dutch roll is damped, the long time period spiral mode begins. If the Dutch roll damps and leaves the aircraft at some small β, then the effect is to induce a rolling moment. If the bank angle gradually increases and the aircraft enters a spiral dive, the mode is unstable. Since the motion is slow (long time period), we may say

$$s_s \simeq \frac{\left(C_{n_r} C_{l_\beta} - C_{n_\beta} C_{l_r}\right) \rho U_o S b^2}{4\left(I_z C_{l_\beta} - I_{xz} C_{n_\beta}\right)}$$

I_{xz} is usually small so $I_{xz} C_{n_\beta}$ can usually be assumed small in comparison with $I_z C_{L_\beta}$. The spiral mode will be stable when the sign of the above root is negative. Since C_{l_β} is negative, the root will be negative as long as

$$C_{l_\beta} C_{n_r} > C_{n_\beta} C_{l_r} \tag{8.28}$$

Note that C_{n_r} and C_{l_β} are both negative while C_{n_β} and C_{l_r} are both positive. Thus, for spiral stability, we must increase the dihedral effect, C_{l_β}, and decrease the weathercock effect, C_{n_β}.

8.5.3.4 Dutch Roll Mode Approximate Equations. For airplanes with relatively small dihedral effect, C_{l_β}, the Dutch roll mode consists primarily of sideslipping and yawing. An approximation to the Dutch roll mode of motion can be obtained from Equation 8.25 by specifying that pure sideslip occurs ($\beta = -\Psi$) and eliminating the rolling degree of freedom. The resulting approximations to Dutch roll damping and natural frequency are:

$$\zeta_{DR} \approx - \frac{bC_{n_r}}{4U_o} \sqrt{\frac{b\rho U_o^{\,2} S}{2C_{n_\beta} I_z}}$$

(8.29)

$$\omega_{n_{DR}} \approx \sqrt{\frac{C_{n_\beta} \quad \rho S b U_o^{\,2}}{2I_z}}$$

In practice, the Dutch roll mode natural frequency is well predicted, but because of the usually large values of C_{l_β} in addition to significant values of I_{xz}, the Dutch roll damping is not well predicted.

If we specify that there are no large changes in yawing moments, Σ = 0) and the Dutch roll mode consists primarily of rolling motion, the resulting approximations to the Dutch roll damping and natural frequency are

$$\zeta_{DR} \approx - C_{y_\beta} \frac{\rho U_o S}{4m} \sqrt{\frac{C_{l_p}}{C_{l_\beta}} \left(\frac{b}{2g} \right)}$$

(8.30)

$$\omega_{n_{DR}} \approx \sqrt{\frac{C_{l_\beta}}{C_{l_p}} \left(\frac{2g}{b} \right)}$$

Just as in the case of Equation 8.29, the Dutch roll damping ratio is not well predicted. To predict the Dutch roll damping ratio, a complete evaluation of Equation 8.25 must be made.

These approximations do give a physical insight into the parameters that affect the Dutch roll mode, and the effect that changes in these parameters such as those caused by configuration changes, stores, and fuel loading may have on flying qualities.

8.5.3.5 Coupled Roll Spiral Mode. This mode of lateral-directional motion has rarely been exhibited by aircraft, but the possibility exists that it can indeed happen. If this mode is present, the characteristic equation for asymmetric motion has two pairs of complex conjugate roots instead of the usual one complex conjugate pair along with two real roots. The phenomenon

occurs as the roll mode root decreases in absolute magnitude while the spiral mode root becomes more negative until they meet and split off the real axis to form a second complex conjugate pair of roots, as depicted in Figure 8.21.

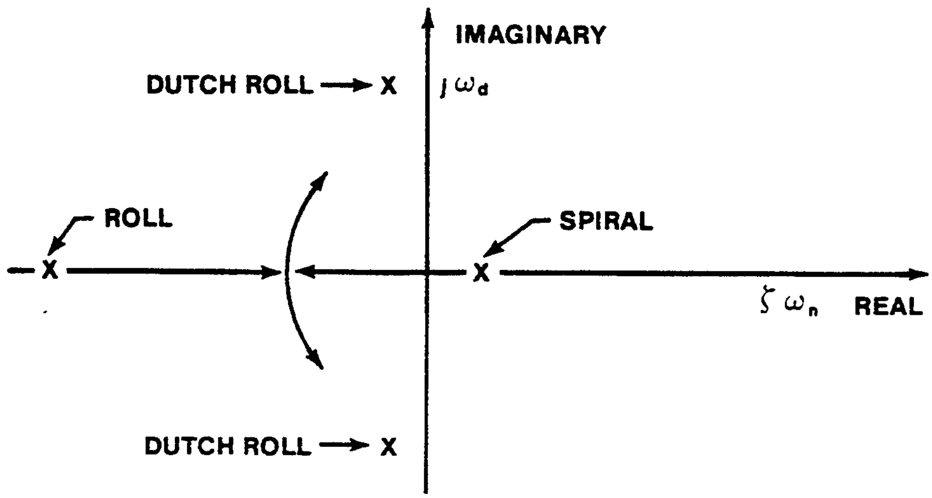


FIGURE 8.21. COUPLED ROLL SPIRAL MODE

At least two in-flight experiences with this mode have been documented and have shown that a coupled roll spiral mode causes significant piloting difficulties. One occurrence involved the M2-F2 lifting body, and a second involved the Flight Dynamics Lab variable-stability NT-33. Some designs of V/STOL aircraft have indicated that these aircraft would exhibit a coupled roll spiral mode in a portion of their flight envelope (Reference 8.3). Some pilot comments from simulator evaluations are "rolly," "requires tightly closed roll control loop," or "will roll on its back if you don't watch it."

A coupled roll spiral mode can result from a high value for C_{l_β} and a low value for C_{l_p}. The M2-F2 lifting body did in fact possess a high dihedral effect and quite a low roll damping. Examination of the equations for the roll mode and spiral mode characteristic equation roots shows how the root locus shown in Figure 8.21 could result as C_{l_p} decreases in absolute magnitude and C_{l_β} increases.

8.6 STABILITY DERIVATIVES

Some of the stability derivatives are particularly pertinent in the study of the dynamic modes of aircraft motion, and the more important ones appearing in the functional equations which characterize the dynamic modes of motion should be understood. C_{M_q}, C_{M_α}, C_{l_p}, C_{l_β}, C_{n_r}, and C_{n_β} are discussed in the following paragraphs.

8.6.1 C_{n_β}

The stability derivative C_{n_β} is the change in yawing moment coefficient with variation in sideslip angle. It is usually referred to as the static directional derivative or the "weathercock" derivative. When the airframe sideslips, the relative wind strikes the airframe obliquely, creating a yawing moment, N, about the center of gravity. The major portion of C_{n_β} comes from the vertical tail, which stabilizes the body of the airframe just as the tail feathers of an arrow stabilize the arrow shaft. The C_{n_β} contribution due to the vertical tail is positive, signifying static directional stability, whereas the C_{n_β} due to body is negative, signifying static directional instability. There is also a contribution to C_{n_β} from the wing, the value of which is usually positive but very small compared to the body and vertical tail contributions.

The derivative C_{n_β} is very important in determining the dynamic lateral stability and control characteristics. Most of the references concerning full-scale flight tests and free-flight wind tunnel model tests agree that C_{n_β} should be as high as possible for good flying qualities. A high value of C_{n_β} aids the pilot in effecting coordinated turns and prevents excessive sideslip and yawing motions in extreme flight maneuvers and in rough air. C_{n_β} primarily determines the natural frequency of the Dutch roll oscillatory mode of the airframe, and it is also a factor in determining the spiral stability characteristics.

8.6.2 C_{n_r}

The stability derivative C_{n_r} is the change in yawing moment coefficient with change in yawing velocity. It is known as the yaw damping derivative. When the airframe is yawing at an angular velocity, r, a yawing moment is produced which opposes the rotation. C_{n_r} is made up of contributions from the wing, the fuselage, and the vertical tail, all of which are negative in sign. The contribution from the vertical tail is by far the largest, usually amounting to about 80% or 90% of the total C_{n_r} of the airframe.

The derivative C_{n_r} is very important in lateral dynamics because it is the main contributor to the damping of the Dutch roll oscillatory mode. It also is important to the spiral mode. For each mode, large negative values of C_{n_r} are desired.

8.6.3 C_{M_α}

This stability derivative is the change in pitching moment coefficient with varying angle of attack and is commonly referred to as the longitudinal static stability derivative. When the angle of attack of the airframe increases from the equilibrium condition, the increased lift on the horizontal tail causes a negative pitching moment about the center of gravity of the airframe. Simultaneously, the increased lift of the wing causes a positive or negative pitching moment, depending on the fore and aft location of the lift vector with respect to the center of gravity. These contributions together with the pitching moment contribution of the fuselage are combined to establish the derivative C_{M_α}. The magnitude and sign of the total C_{M_α} for a particular airframe configuration are thus a function of the center of gravity position, and this fact is very important in longitudinal stability and control. If the center of gravity is ahead of the neutral point, the value of C_{M_α} is negative, and the airframe is said to possess static longitudinal

stability. Conversely, if the center of gravity is aft of the neutral point, the value of C_{M_α} is positive, and the airframe is then statically unstable. C_{M_α} is perhaps the most important derivative as far as longitudinal stability and control are concerned. It primarily establishes the natural frequency of the short period mode and is a major factor in determining the response of the airframe to elevator motions and to gusts. In general, a large negative value of C_{M_α} (i.e., large static stability) is desirable for good flying qualities. However, if it is too large, the required elevator effectiveness for satisfactory control may become unreasonably high. A compromise is thus necessary in selecting a design range for C_{M_α}. Design values of static stability are usually expressed not in terms of C_{M_α} but rather in terms of the derivative $C_{M_{C_L}}$, where the relation is $C_{M_\alpha} = C_{M_{C_L}} C_{L_\alpha}$. It should be pointed out that $C_{M_{C_L}}$ in the above expression is actually a partial derivative, for which the forward speed remains constant.

8.6.4 C_{M_q}

The stability derivative C_{M_q} is the change in pitching moment coefficient with varying pitch velocity and is commonly referred to as the pitch damping derivative. As the airframe pitches about its center of gravity, the angle of attack of the horizontal tail changes and lift develops on the horizontal tail, producing a negative pitching moment on the airframe and hence a contribution to the derivative C_{M_q}. There is also a contribution to C_{M_q} because of various "dead weight" aeroelastic effects. Since the airframe is moving in a curved flight path due to its pitching, a centrifugal force is developed on all the components of the airframe. The force can cause the wing to twist as a result of the dead weight moment of overhanging nacelles and can cause the horizontal tail angle of attack to change as a result of fuselage bending due to the weight of the tail section. In low speed flight, C_{M_q} comes mostly from the effect of the curved flight path on the horizontal tail, and its sign is negative. In high speed flight the sign of

C_{M_q} can be positive or negative, depending on the nature of the aeroelastic effects. The derivative C_{M_q} is very important in longitudinal dynamics because it contributes a major portion of the damping of the short period mode for conventional aircraft. As pointed out, this damping effect comes mostly from the horizontal tail. For tailless aircraft, the magnitude of C_{M_q} is consequently small; this is the main reason for the usually poor damping of this type of configuration. C_{M_q} is also involved to a certain extent in phugoid damping. In almost all cases, high negative values of C_{M_q} are desired.

8.6.5 C_{l_β}

This stability derivative is the change in rolling moment coefficient with variation in sideslip angle and is usually referred to as the "effective dihedral derivative." When the airframe sideslips, a rolling moment is developed because of the dihedral effect of the wing and because of the usual high position of the vertical tail relative to the equilibrium x-axis. No general statements can be made concerning the relative magnitude of the contributions to C_{l_β} from the vertical tail and from the wing since these contributions vary considerably from airframe to airframe and for different angles of attack of the same airframe. C_{l_β} is nearly always negative in sign, signifying a negative rolling moment for a positive sideslip.

C_{l_β} is very important in lateral stability and control, and is therefore usually considered in the preliminary design of an airframe. It is involved in damping both the Dutch roll mode and the spiral mode. It is also involved in the maneuvering characteristics of an airframe, especially with regard to lateral control with the rudder alone near stall.

8.6.6 C_{l_p}

The stability derivative C_{l_p} is the change in rolling moment coefficient with change in rolling velocity and is usually known as the roll damping derivative. When the airframe rolls at an angular velocity p, a rolling moment is produced as a result of this velocity; this moment opposes the rotation. C_{l_p} is composed of contributions, negative in sign, from the wing and the horizontal and vertical tails. However, unless the size of the tail is unusually large in comparison with the size of the wing, the major portion of the total C_{l_p} comes from the wing.

The derivative C_{l_p} is quite important in lateral dynamics because essentially C_{l_p} alone determines the damping in roll characteristics of the aircraft. Normally, it appears that small negative values of C_{l_p} are more desirable than large ones because the airframe will respond better to a given aileron input and will suffer fewer flight perturbations due to gust inputs.

8.7 DYNAMIC STABILITY FLIGHT TESTS

The dynamic response of an aircraft to various pilot control inputs is important in evaluating its handling qualities. The aircraft may be statically stable, yet its dynamic response could be such that a dangerous flight characteristic results. The aircraft must have dynamic qualities that permit the design mission to be accomplished.

The purpose of the dynamic stability flight test is to investigate an aircraft's primary modes of motion. An airplane usually has five major modes of free motion: phugoid, short period, rolling, Dutch roll and spiral. Flight test determines the acceptability of these modes - frequency, damping, and time constant being the characteristics of primary importance.

There are several different forms that the modes of motion may take. Figure 8.22 shows four possibilities for aircraft free motion: a pure divergence, a pure convergence, an undamped oscillation, or negatively damped oscillation. The aircraft being a rather complicated dynamic system, will move in a manner that is a combination of several different modes at the same time. One of the problems of flight testing is to excite each individual mode independently.

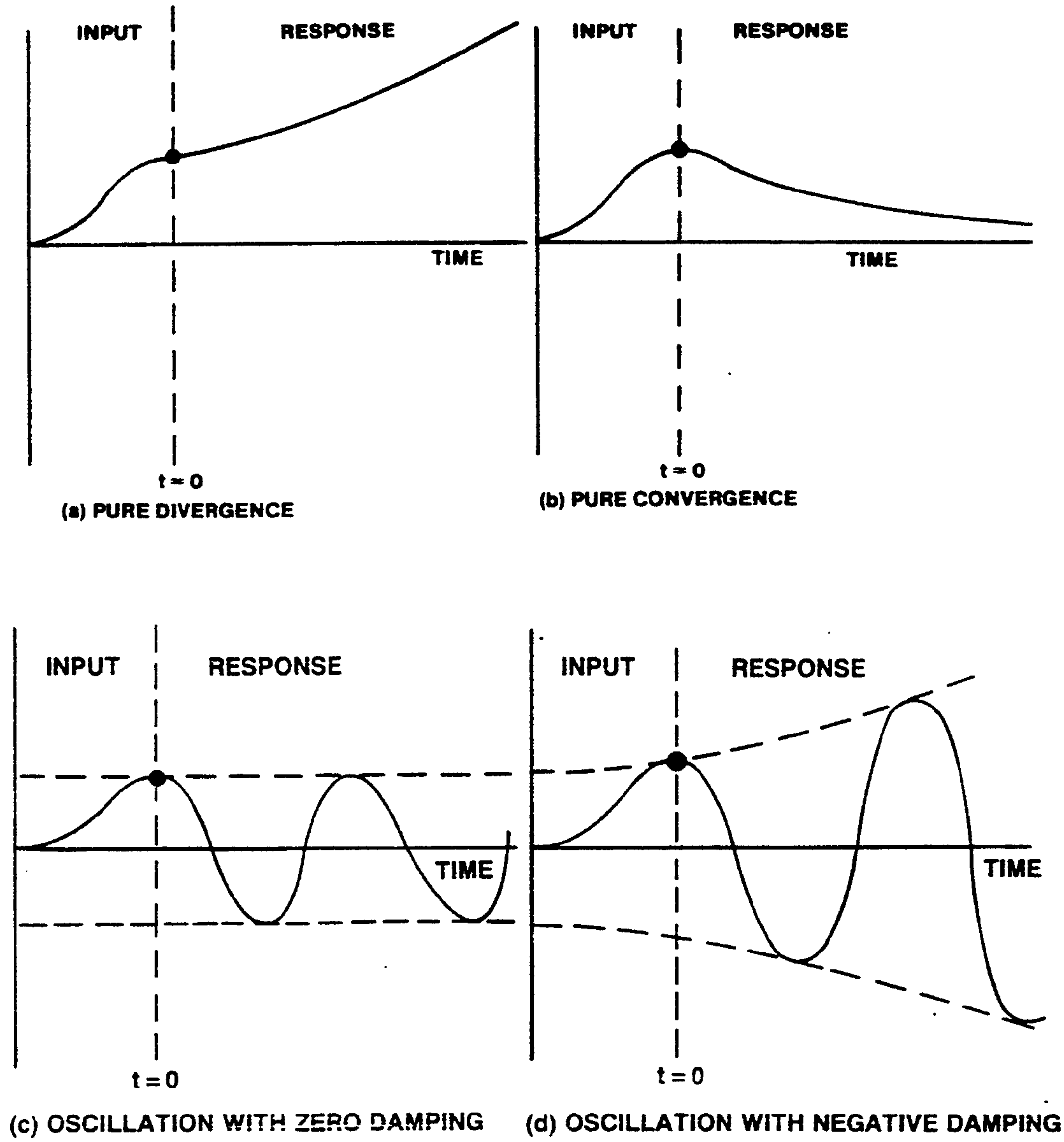


FIGURE 8.22 AIRCRAFT FREE MOTION POSSIBILITIES

8.7.1 Control Inputs

There are several different control inputs that could be used to excite the dynamic modes of motion of an aircraft. To accomplish the task of obtaining the free response of an aircraft, the pilot makes an appropriate control input, removes himself from the loop, and observes the resulting aircraft motion. Three inputs that are frequently used in stability and control investigations are: the step input, the singlet, and the doublet.

8.7.1.1 Step Input. When a step input is made, the applicable control is rapidly moved to a desired new position and held there. The aircraft motion resulting from this suddenly applied new control position is recorded for analysis. A mathematical representation of a step input assumes the deflection occurs in zero time and is contrasted to a typical actual control position time history in Figure 8.23. The "unit step" input is frequently used in theoretical analysis and has the magnitude of one radian, which is equivalent to 57.3°. Specifying control inputs in dimensionless radians instead of degrees is convenient for use in the non-dimensional equations of motion.

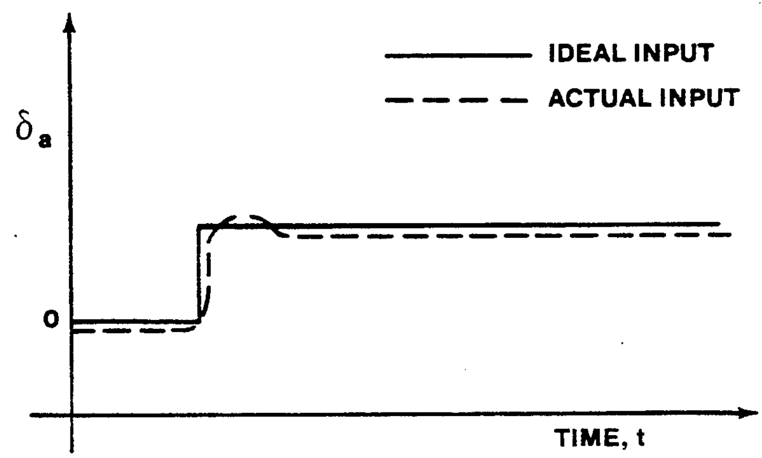


FIGURE 8.23 STEP INPUT

8.7.1.2 Pulse. When a pulse, or singlet is applied, the control is moved to a desired position, held momentarily, and then rapidly returned to its original position. The pilot can then remove himself from the loop and observe the free aircraft response. Again, deflections are theoretically assumed to occur instantaneously. An example of a pulse, or singlet, is shown in Figure 8.24.

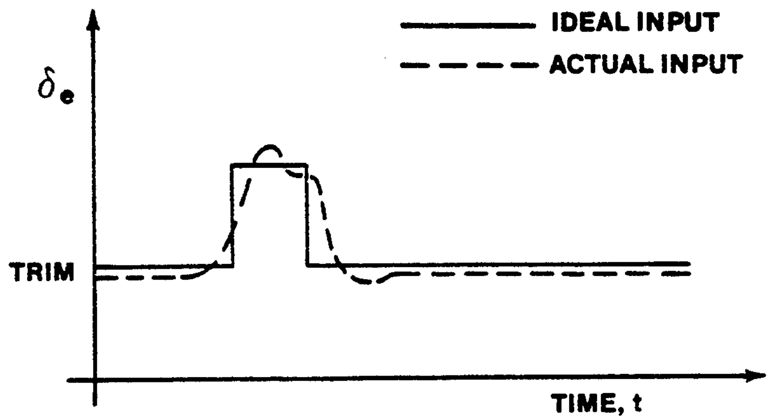


FIGURE 8.24 PULSE INPUT

The "unit impulse" is frequently used in theoretical analysis and is related to the pulse input. The unit impulse is the mathematical result of a limiting process which has an infinitely large magnitude input applied in zero time and an area of unity.

8.7.1.3 Doublet. A doublet is a double pulse which is skew symmetric with time. After exciting a dynamic mode of motion with this input and removing himself from the control loop, the pilot can record the aircraft open loop motion (Figure 8.25).

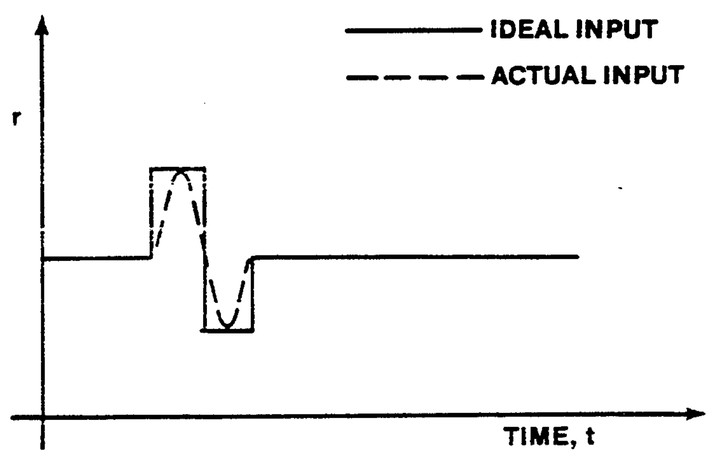


FIGURE 8.25 DOUBLET INPUT

8.7.2 Pilot Estimation Of Second Order Response

Pilot-observed data can be used to obtain approximate values for the damped frequency and damping ratio for second order motion such as the short period or Dutch roll.

To obtain a value for ω_d, the pilot needs merely to observe the number of cycles that occur during a particular increment of time.

Then,

$$f_d = \frac{\text{Number of Cycles}}{\text{Time Increment}} = \text{cycles/sec} \tag{8.31}$$

And

$$\omega_d = f_d \frac{\text{cycles}}{\text{sec}} \times \frac{2\pi \text{ radians}}{\text{cycle}} = \text{radians/sec} \tag{8.32}$$

The number of cycles can be estimated either by counting overshoots (peaks) or zeroes of the appropriate variable. For short period motion, perturbed θ is easily observed, and if counting overshoots is applied to the motion shown in Figure 8.26, the result is

$$f_d = \frac{1}{2} \frac{(\text{Number of Peaks} - 1)}{(\text{Time Increment})}$$

$$f_d = \frac{\frac{1}{2}(4-1)}{3} = .5 \text{ cycles/sec}$$

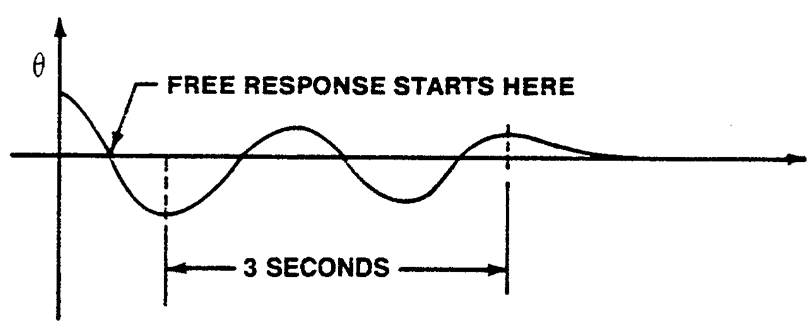


FIGURE 8.26 SECOND ORDER MOTION

If zeroes are counted, then

$$f_d = \frac{\frac{1}{2}\text{ (number of zeroes - 1)}}{\text{(Time Increment)}} \text{ cycles/sec}$$

The pilot can obtain an estimated value for ζ by noting the number of peaks that exist during second order motion and using the approximation

$$\zeta \simeq \frac{1}{10}\ \text{(7 - Number of Peaks)} \qquad (8.33)$$

for $.1 < \zeta < .7$

The motion shown in Figure 8.26 thus has an approximate value.

$$\zeta \simeq \frac{1}{10}\ (7 - 4) = .3$$

Note that the peaks which occur during aircraft free response are the ones to be used in Equation 8.26. If zero observable peaks exist during a second order motion, the best estimate for the value of ζ is then "heavily damped, .7 or greater." If seven or more peaks are observed, the best estimate for the value of ζ is "lightly damped, .1 or less."

8.7.3 Short Period Mode

The short period is characterized by pitch angle, pitch rate, and angle of attack change while essentially at constant airspeed and altitude. The short period mode is an important flying quality because its period can approach the limit of pilot reaction time and it is the mode which a pilot uses for longitudinal maneuvers in normal flying. The period and damping may be such that the pilot may induce an unstable oscillation if he attempts to damp the motion with control movements. Hence, heavy damping of this mode is

desirable. Although heavy damping of the short period is desired, investigations have shown that damping alone is insufficient for good flying qualities. In fact, very high damping may result in poor handling qualities. It is the combination of damping and frequency of the motion that is important.

8.7.3.1 Short Period Flight Test Technique. To examine the short period mode, stabilize the airplane at the desired flight condition (altitude, airspeed, normal acceleration). Trim the control forces to zero (for one g normal acceleration) and start recording. Abruptly deflect the longitudinal control to obtain a change in normal acceleration of about one-half g. A suggested technique is to apply a longitudinal control doublet (a small positive displacement followed immediately by a negative displacement of the same magnitude followed by rapidly returning the control to the trimmed position). For stick-fixed stability, return the control to neutral and hold fixed. For stick-free stability release the control after it is returned to neutral.

The abruptness and magnitude of the control input must be approached with due care! Use very small inputs until it is determined that the response is not violent. Start with small magnitudes and gradually work up to the desired excitation. When the aircraft transient motion stops, stop recording data. An input that is too sharp or too large could very easily excite the aircraft structural mode or produce a flutter that might seriously damage the airplane and/or injure the pilot. If the aircraft is equipped with artificial stabilization devices, the test should be conducted with this device off as well as on.

8.7.3.2 Short Period Data Required. The trim conditions of pressure altitude, airspeed, weight, cg position, and configuration should be recorded. The test variables of concern are: airspeed, altitude, angle of attack, normal acceleration, pitch angle, pitch rate, control surface position, and control stick/yoke position.

8.7.3.3 Short Period Data Reduction. Short period mode investigations have shown that frequency as well as damping is important in a consideration of flying qualities. This is so because at a given frequency, damping alters the phase angle of the closed-loop system (which consists of a pilot coupled to the airframe system). Phase angle of the total system governs the dynamic stability.

The short period frequency, damping, and n/α can be determined from trace of the time history of aircraft response to a pitch doublet as shown in Figure 8.27.

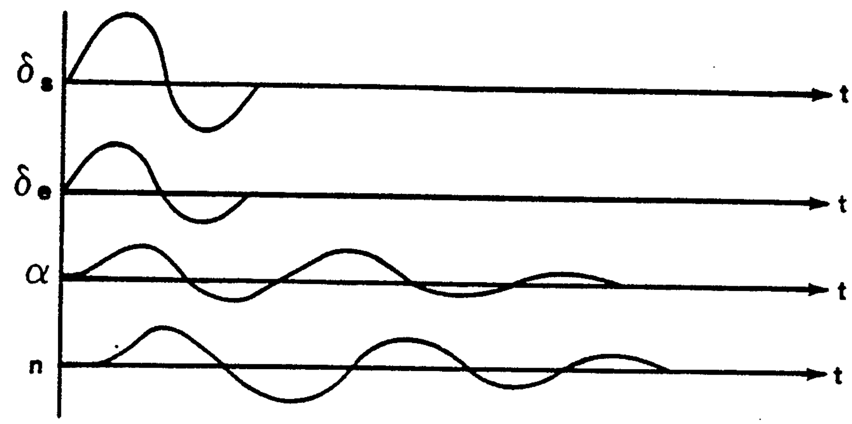


FIGURE 8.27 SHORT PERIOD RESPONSE

The MIL-STD-1797 specifies that the angle of attack trace should be used to determine the short period response frequency and damping ratio; however, load factor, pitch angle, and pitch rate are all indicators of the same short period free response characteristics. Either the Log Decrement or Time Ratio data reduction methods can be applied to the short period response trace.

8.7.3.3.1 Log Decrement Method. If the short period response is oscillatory and the damping ratio is .4 or less (three or more overshoots), proceed with the log decrement method of data reduction. This method is also called the subsidence ratio or the transient peak-ratio method.

Using the angle of attack trace, draw a mean value line at the steady-state trimmed angle of attack. Measure the values of each peak deviation from trimmed angle of attack for ΔX_1, ΔX_2, ΔX_3, etc., as shown in Figure 8.28.

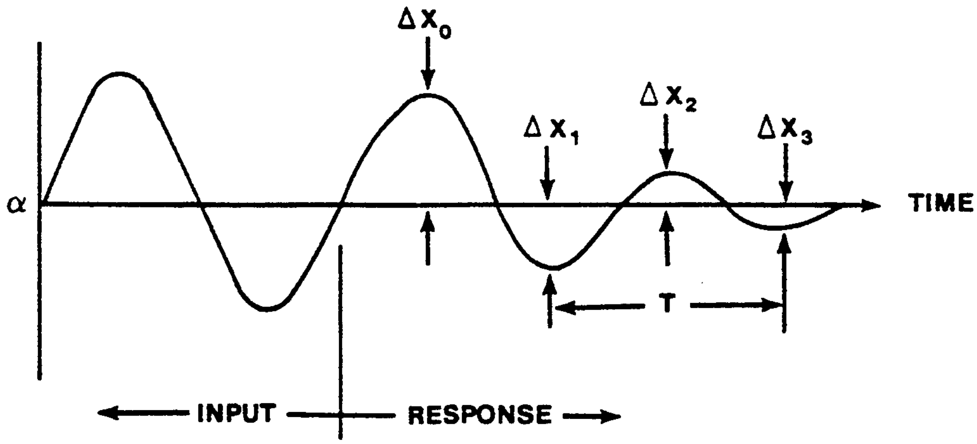


FIGURE 8.28 SUBSIDENCE RATIO ANALYSIS

Determine the transient peak ratios, $\Delta X_1/\Delta X_0$, $\Delta X_2/\Delta X_1$, $\Delta X_3/\Delta X_2$. Enter Figure 8.30 with the transient peak ratio values for $\Delta X_1/\Delta X_0$, $\Delta X_2/\Delta X_1$, $\Delta X_3/\Delta X_2$ and determine corresponding values for damping ratios ζ_1, ζ_2, ζ_3. The average of the damping ratios will yield the value of the overall short period damping ratio. For very lightly damped oscillatory response, Figure 8.31 can be used to determine damping ratio. The m = 1 line is used when comparing peak ratios $\Delta X_1/\Delta X_0$, $\Delta X_2/\Delta X_1$; the m = 2 line is used for peak ratios of $\Delta X_2/\Delta X_0$, $\Delta X_3/\Delta X_1$, etc.

The period, T, of the short period response can be determined by measuring the time between peaks as shown in Figure 8.28. The short period damped frequency is then calculated by $\omega_d = 2\pi/T$ (rad/sec). The short period natural frequency is computed using $\omega_n = \omega_d/\ 1 - \zeta^2$.

8.7.3.3.2 Time Ratio Method. If the damping ratio is between .5 and 1.5 (two or less overshoots), then the time ratio method of a data reduction can be used to determine short period response frequency and damping ratio.

Select a peak on the angle of attack trace where the response is free. Divide the amplitude of the peak into the values of .736, .409, and .199. Measure time values t_1, t_2, and t_3 as shown in Figure 8.29. Form the time ratios t_2/t_1, t_3/t_1, and $(t_3 - t_2)/(t_2 - t_1)$.

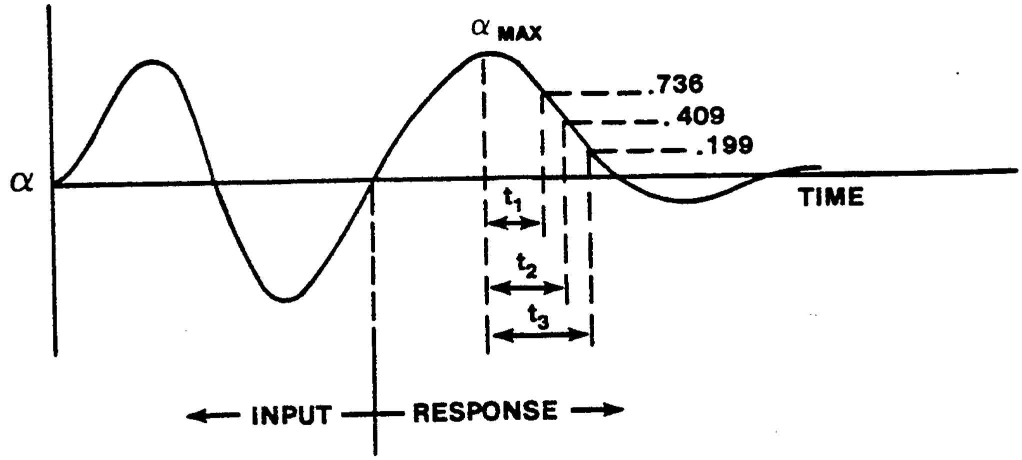


FIGURE 8.29 TIME RATIO ANALYSIS

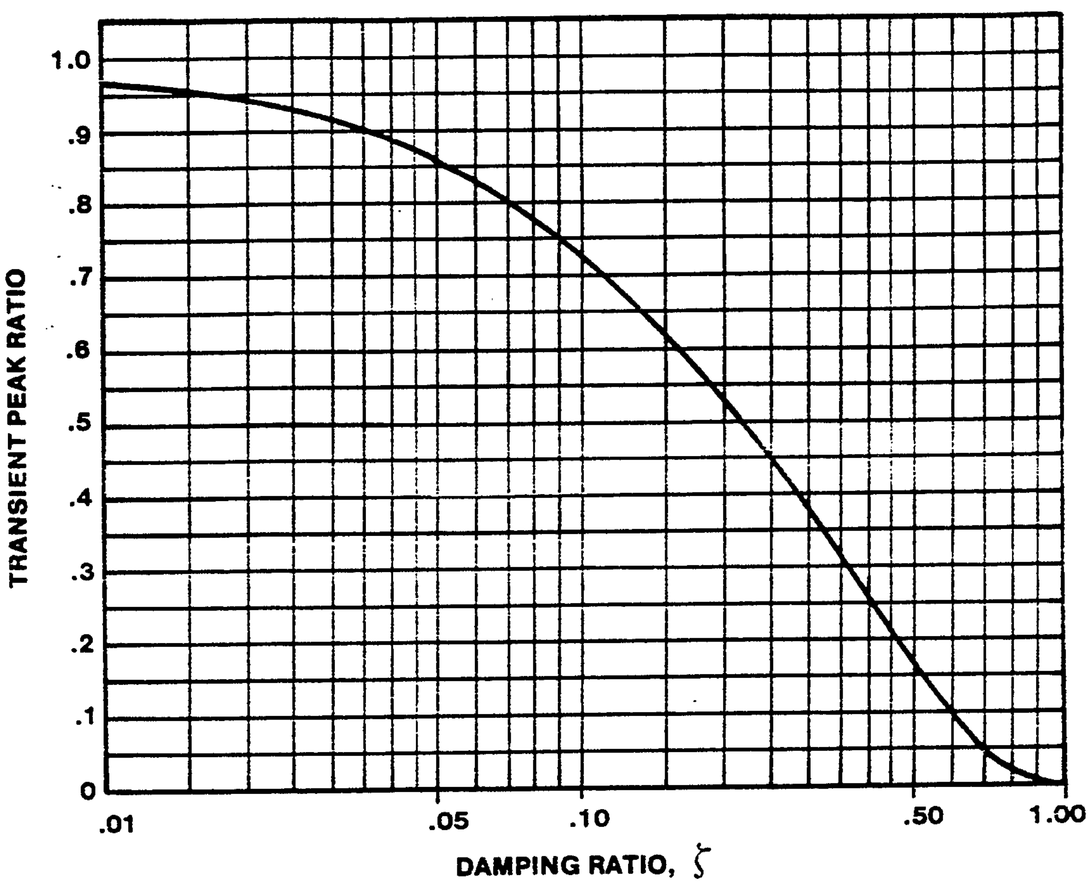


FIGURE 8.30 DETERMINING ζ BY TRANSIENT-PEAK-RATIO METHOD

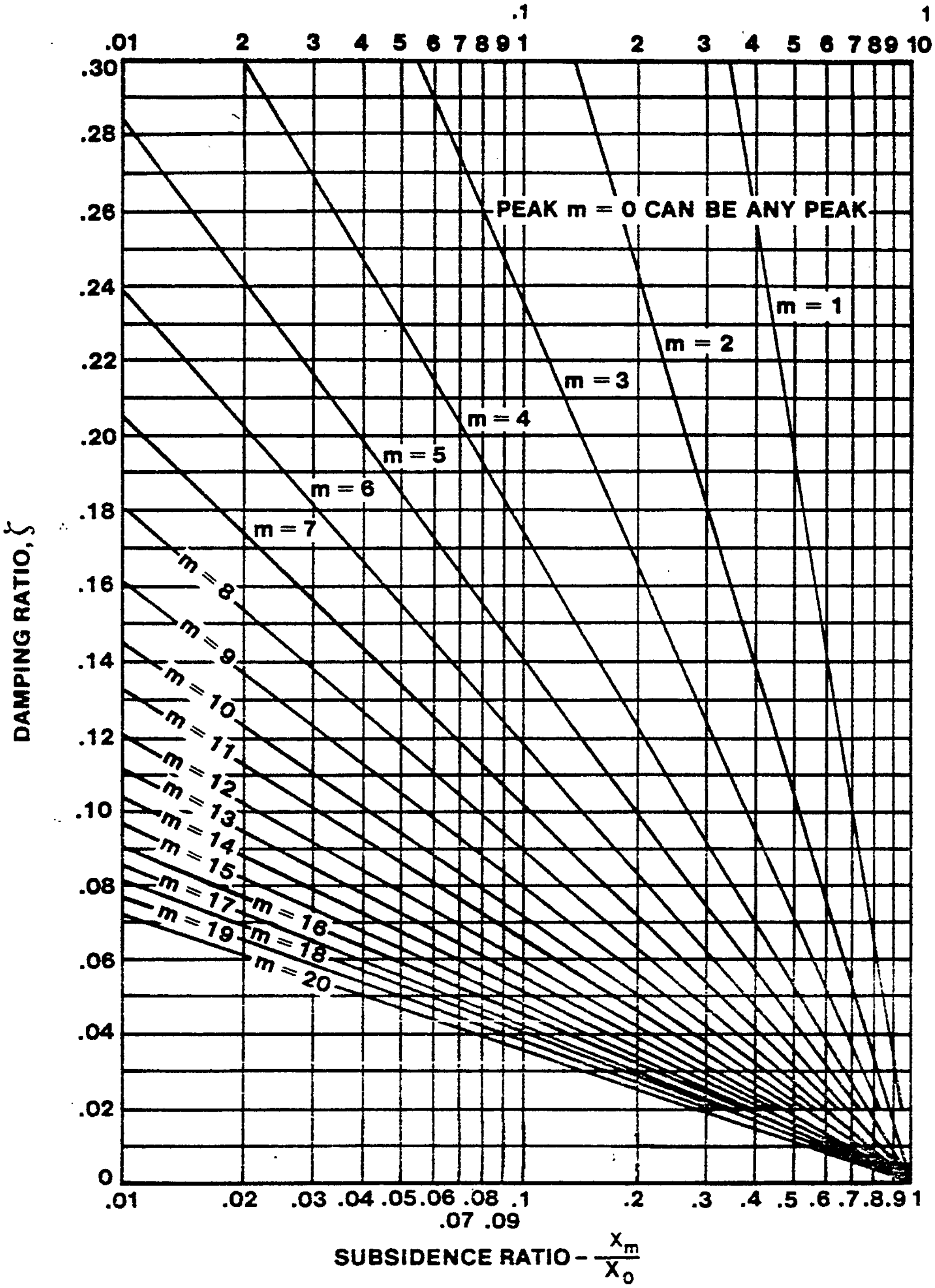


FIGURE 8.31 DAMPING RATIO AS A FUNCTION OF SUBSIDENCE RATIO

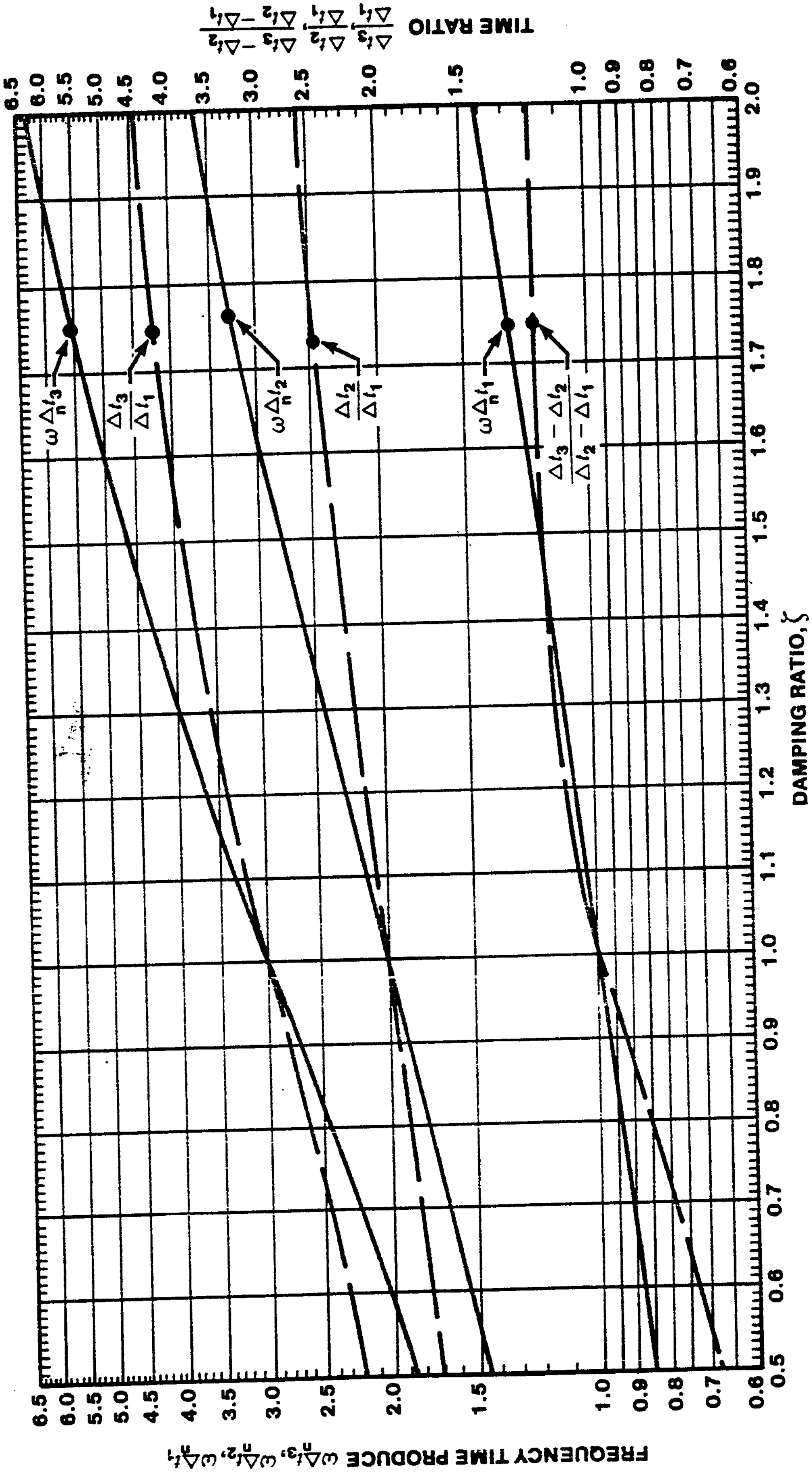


FIGURE 8.32 DETERMINING ζ AND ω_n BY TIME-RATIO METHOD

Enter Figure 8.32 at the Time Ratio side and find the corresponding damping ratio for each time ratio. Average these damping ratios to determine the short period damping ratio, ζ_{sp}.

Re-enter Figure 8.32 with the average short period damping ratio and find the frequency time products $\omega_n t_1$, $\omega_n t_2$, and $\omega_n t_3$. To determine the natural frequency, ω_n, compute:

$$\omega_n = \frac{\omega_n t_1}{t_1} \qquad \omega_n = \frac{\omega_n t_2}{t_2} \qquad \omega_n = \frac{\omega_n t_3}{t_3}$$

Average these natural frequencies to determine the overall short period natural frequency.

8.7.3.4 <u>n/α Data Reduction</u>. From the time history traces of load factor and angle of attack free response, determine the peak value of angle of attack that produced the peak load factor, n, as shown in Figure 8.33. Compute the ratio $\Delta n/\Delta\alpha$.

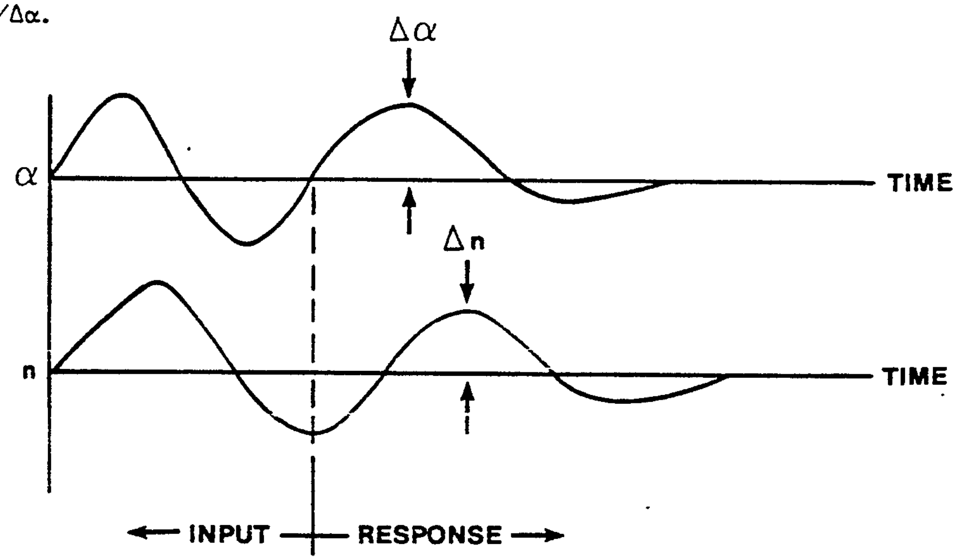


FIGURE 8.33 n/α ANALYSIS

8.7.3.5 <u>Short Period Mil-Std Requirements</u>. MIL-STD-1797 specifies that an aircraft's short period response, controls fixed and free, shall meet the requirements of frequency, damping and acceleration sensitivity established in Appendix A Paragraph 4.2.1.2. Residual oscillations shall not be greater than .02g, (Appendix A Para 4.2.3) at the pilot's station.

Tests for short period stability should be conducted from level flight at several altitudes and Mach. Closed loop short period stability tests should also be made at various normal accelerations in maneuvering flight. This stability, when coupled to the pilot, is especially important to tracking and formation flying.

8.7.4 Phugoid Mode

The phugoid mode is generally not considered an important flying quality because its period is usually of sufficient duration that the pilot has little difficulty controlling it. However, under certain conditions it is possible for the damping to degenerate sufficiently so that the phugoid mode becomes important. The phugoid is characterized by airspeed, altitude, pitch angle, and rate variations while at essentially constant angle of attack.

8.7.4.1 Phugoid Flight Test Technique. The phugoid mode may be examined by stabilizing the airplane at the desired flight conditions and trimming the control forces to zero. Smoothly increase the pitch angle until the airspeed reduces 10 to 15 knots below the trim airspeed and return the nose to the trimmed attitude. For stick-fixed stability return the control to neutral and then release it. After the control is released or returned, it may be necessary to maintain wings level by light lateral or slight rudder pressure. Damping and frequency of phugoid motion may be changed appreciably by the presence of small bank angles (5° to 15°). It may be very difficult to return the control to its trimmed position if the aircraft control system has a very large friction band. In such a case, the airspeed increment may be obtained by an increase or decrease in power and by returning it to its trim setting or extending a drag device. In either case the aircraft configuration should be that of the trim condition at the time the data measurements are made.

8.7.4.2 Phugoid Data Required. The trim conditions of pressure altitude, airspeed, weight, cg position and configuration should be recorded.

The damping can be determined by hand recording the maximum and minimum airspeed excursions during at least two cycles of the phugoid free response. In addition, the period can be accurately hand recorded by noting the time between zero vertical velocity points.

8.7.4.3 <u>Phugoid Data Reduction</u>. To determine the phugoid damping ratio (ζ), sketch the damping envelope on the working plot of airspeed versus time. Measure the width of the envelope at the peak values of the oscillation. Form the subsidence ratios (X_m/X_0). Find the damping ratio for each subsidence ratio from Figure 8.30 or 8.31. Average these damping ratios. If the subsidence ratio is greater than 1.0, then use the inverse of that subsidence ratio. The damping ratio thus determined will be negative, and the mode divergent.

Another method of determining phugoid damping ratio analogous to the above subsidence ratio method is to compute the difference between successive maximum and minimum velocities and assign these magnitudes as ΔX_0, ΔX_1, ΔX_2, etc., as shown in Figure 8.34. Next, form the transient peak ratios $\Delta X_1/\Delta X_0$, $\Delta X_2/\Delta X_1$ and find the damping ratio from Figure 8.30 or Figure 8.31.

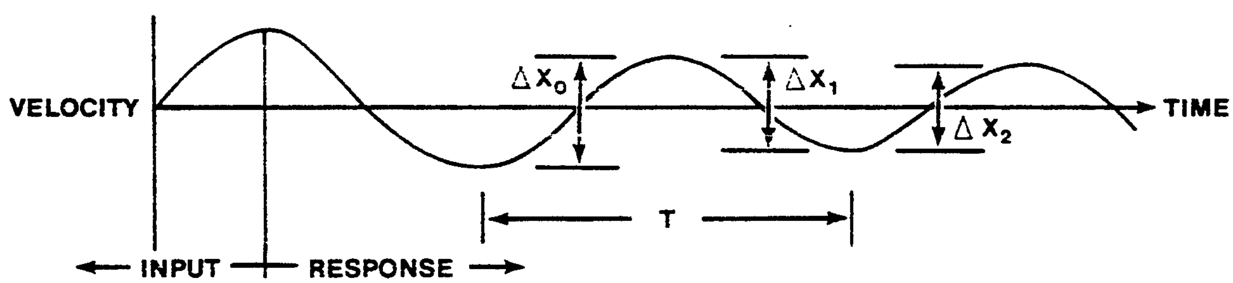


FIGURE 8.34 PHUGOID TRANSIENT PEAK RATIO ANALYSIS

The damped frequency of the phugoid can be determined from the hand recorded period by $\omega_d = 2\pi/T$ (rad/sec). The natural frequency is then computed by

$$\omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}}$$

For highly augmented aircraft, the equivalent damping is to be determined from the three degree-of-freedom transfer function Θ/δ_{es} or θ/F_{es}. The level 3 T_2 requirement is to be checked from the time response of the actual aircraft for both nose-up and nose-down control inputs. This method is discussed in Appendix A para 4.2.1.1.

8.7.4.4 Long Term Pitch Response MIL-STD Requirement (Phugoid). The MIL-STD-1797 requirement for phugoid damping is outlined in Appendix A Paragraph 4.2.1.1.

8.7.5 Dutch Roll Mode

The Dutch roll lateral-directional oscillations involve roll, yaw, and sideslip. The stability of the Dutch roll mode varies with airplane configuration, angle of attack, Mach, and damper configuration. The presence of a lightly damped oscillation adversely affects aiming accuracy during bombing runs, firing of guns and rockets, and precise formation work such as in-flight refueling.

Stability of the oscillations is represented by the damping ratio; however, the frequency of an oscillation and the ϕ/β ratio are also important in order to correlate the motion data with the pilot's opinion of handling qualities. If the frequency is higher than pilot reaction time, the pilot cannot control the oscillation and in some cases may reinforce the oscillation to an undesirable amplitude. Since it is the damping frequency combination which influences pilot opinion more than damping alone, some effort should be made to correlate this combination with pilot opinion of the lateral-directional oscillation.

At supersonic speeds, directional stability often decreases with increased Mach and altitude for constant g. An evaluation should proceed cautiously to avoid possible divergent responses that can result from nonlinear aerodynamics.

8.7.5.1 Dutch Roll Flight Test Techniques.

8.7.5.1.1 Rudder Pulse (doublet). Stabilize the airplane in level flight at test flight conditions and trim. Rapidly depress the rudder in each direction and neutralize. Hold at neutral for control-fixed or release rudder for control free response. For aircraft which require excessive rudder force in some flight conditions, the rudder pulse may be applied through the augmented directional flight control system.

8.7.5.1.2 Release from Steady Sideslip. Stabilize the airplane in level flight at test flight conditions and trim forces to zero. Establish a steady straight-path sideslip angle. Rapidly neutralize controls. Either hold controls for control-fixed or release controls for control-free response. Start with small sideslip in case the aircraft diverges.

8.7.5.1.3 Aileron Pulse. Stabilize the airplane in level flight at test flight conditions and trim. Hold aircraft in a steady turn of 10° to 30° of bank. Roll level at a maximum rate reducing the roll rate to zero at level flight. CAUTION. . . Such a test procedure must be monitored by an engineer who is thoroughly familiar with the inertial coupling of that aircraft and its effect upon structural loads and nonlinear stability.

Nonlinearities in the aircraft response may hinder the extraction of the necessary parameters. These can be induced by large input conditions. Small inputs balanced with instrument sensitivity give the best result.

8.7.5.2 Dutch Roll Data Required. For trim condition, pressure altitude, airspeed, weight, cg position, and aircraft configuration should be recorded. The test variables of concern are bank angle, sideslip angle, yaw rate, roll rate, control positions, and control surface positions.

Flight test data will be obtained as time histories. When determining the damping ratio, the roll rate parameter usually presents the best trace. In addition, the bank angle and sideslip angle time histories will be required to determine the ϕ/β ratio.

8.7.5.3 Dutch Roll Data Reduction. The Dutch roll frequency and damping ratio can be determined from either a bank angle, sideslip angle, or roll rate response time history trace. The roll rate generally gives the best trace for data reduction purposes.

The methods for determining Dutch roll frequency and damping ratio are the same as used for short period data reduction. If the damping ratio is between .5 and 1.5 (2 or less overshoots), then the time ratio method can be employed. For damping ratio of .4 or less (3 or more overshoots), then subsidence ratio methods are applicable for determining Dutch roll frequencies and damping ratio.

The ϕ/β ratio at the test condition can be determined from the ratio of magnitudes of roll angle envelope to sideslip angle envelope at any specified instant of time during the free response motion as shown in Figure 8.35.

For highly augmented aircraft, the Dutch Roll frequency and damping will be determined by matching the actual aircraft response to a lower order form of $\frac{\beta}{F_{rp}}$, over the frequency range from 0.1 rad/sec to 10 rad/sec. If $|\Theta/\beta|$ is large, an alternative match can be done using a lower from of ϕ/δ_{as}, ϕ/F_{as}, β/δ_{as} or F/δ_{as}. These lower forms are given in Appendix A Paragraph 4.6.1.1.

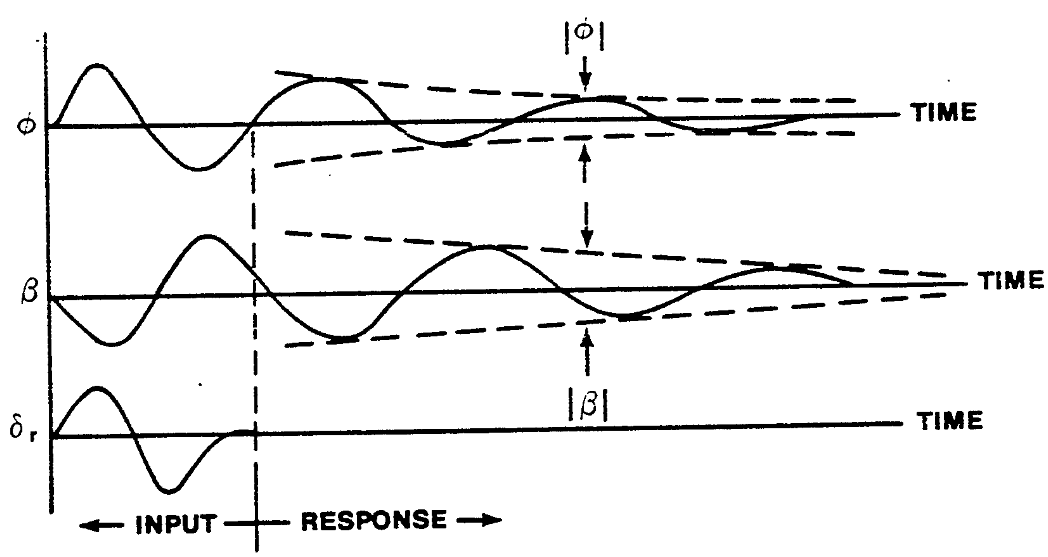


FIGURE 8.35. DETERMINATION OF $|\phi|$ / $|\beta|$

8.7.5.4 Dynamic Lateral - Directional Response Requirements

MIL-STD-1797 requirements for Dutch roll frequency and damping ratio are specified in Appendix A Paragraph 4.6.1.1.

8.7.6 Spiral Mode

The spiral mode is relatively unimportant as a flying quality. However, a combination of spiral instability and lack of precise lateral trimmability may be annoying to the pilot. This problem will be evaluated as a whole due to the difficulty in separating the effects.

The divergent motion is non-oscillatory and is most noticeable in the bank and yaw responses. If an airplane is spirally divergent, it will, when disturbed and not checked, go into a tightening spiral dive. This divergence can be easily controlled by the pilot if the divergence is not too fast.

Excitation of only the spiral mode is difficult because of its relatively large time constant. Any practical input using control surfaces would usually excite other modes as well. If a deficiency in lateral trim control exists, it is often difficult to determine what portion of the resultant motion following a disturbance is caused by the spiral mode. Flight test is used to determine if a combined problem of lateral trim and spiral stability exists. If test results show a definite divergence in hands-off flight, the problem exists.

Spiral divergence is of little importance as a flying quality because it is well within the control capability of the pilot. The ability to maintain lateral trim in hands-off flight for 10 to 20 seconds is important.

8.7.6.1 Spiral Mode Flight Test Technique. Trim the aircraft for hands-off flight, paying particular attention to lateral control and centering the ball. Roll into a 20° bank in one direction, release the controls and measure the bank angle after 20 seconds. Repeat the maneuver in a bank to the opposite side.

8.7.6.2 Spiral Mode Data Required. Record aircraft configuration, weight, cg position, altitude and airspeed. The test variables are bank angle, sideslip angle, control position, and control surface position.

8.7.6.3 Spiral Mode Data Reduction. Average the time to double amplitude for right and left banks at each test condition. Figure 8.36 illustrates bank angle data for spiral mode analysis. For highly augmented aircraft, values of the equivalent spiral time constant is obtained from a fit of the P/F_{as} transfer function. This method is described for the Roll Mode in Appendix A Paragraph 4.5.1.1.

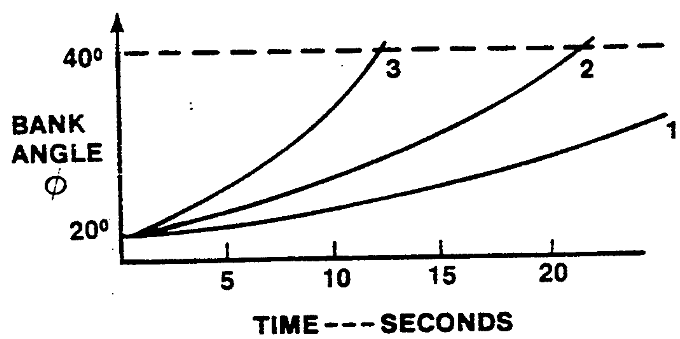


FIGURE 8.36. SPIRAL MODE ANALYSIS

8.7.6.4 Spiral Stability Mil- Std Requirement. Spiral stability is specified in MIL-STD-1797 in Table XXII. This table established minimum times to double amplitude when the aircraft is put into a bank up to 20°, and the controls are freed.

8.7.7 Roll Mode

The roll mode is the primary method that the pilot uses in controlling the lateral attitude of an aircraft. The roll mode represents an aperiodic (nonoscillatory) response to a pilot's lateral stick input which involves almost a pure roll about the x-axis.

Of primary concern to all pilots is the roll performance involving the time required for the aircraft to accelerate to and reach a steady state roll rate in response to a pilot's lateral input. The roll performance parameter is useful in describing the roll response of an airplane in roll mode time constant, τ_R. Physically, τ_R, is that time for which the airplane has reached 63% of its steady-state roll rate following a step input of the ailerons. The roll mode time constant directly influences the pilot's opinion of the maneuvering capabilities of an airplane. In addition, τ_R can affect the piloting technique used in bank angle control tasks.

8.7.7.1 Roll Mode Flight Test Technique. Trim the aircraft for hands-off flight. Roll into a 45° bank in one direction and stabilize the aircraft. Abruptly apply a small step aileron input and hold throughout 90° of bank angle change. The size of the step aileron input should be sufficiently small to allow the aircraft to achieve steady-state roll rate prior to the 90° bank angle change; however, sufficiently large to measure the roll rate with the instrumentation system onboard the aircraft.

8.7.7.2 Roll Mode Data Required. Record aircraft configuration, weight, cg position, lateral fuel loading, attitude and airspeed. The test variables are bank, roll rate, control position and control surface position.

8.7.7.3 Roll Mode Data Reduction. The roll mode time constant, τ_R, can be measured from a time history trace of the roll rate response (Figure 8.37B) or bank angle response (Figure 8.37A) to a step aileron input. From the roll rate, p, trace the time constant, τ_R, can be measured as the time for the roll rate to achieve 63% of the steady-state roll rate response (Figure 8.37A). τ_R can also be determined from the bank angle, ϕ, trace as the time for the extension of the linear slope of the ϕ trace to intersect the initial ϕ axis as represented in Figure 8.37A. It is important to note that the roll mode time constant is independent of the size of the step aileron input. For highly augmented aircraft, an equivalent roll mode time constant is determined by a fit of the ϕ/δ_{as} or ϕ/F_{as}; or β/δ_{rp} or β/F_{rp} transfer functions to the actual aircraft response over a frequency range from 0.1 rad/sec to 10 rad/sec. These transfer functions are specified in Appendix A Paragraph 4.5.1.1.

8.7.7.4 Roll Mode Mil- Std Requirements. The roll mode requirements are specified in MIL-STD-1797, Table VII. This table establishes limits on the maximum allowable time for the roll mode time constant.

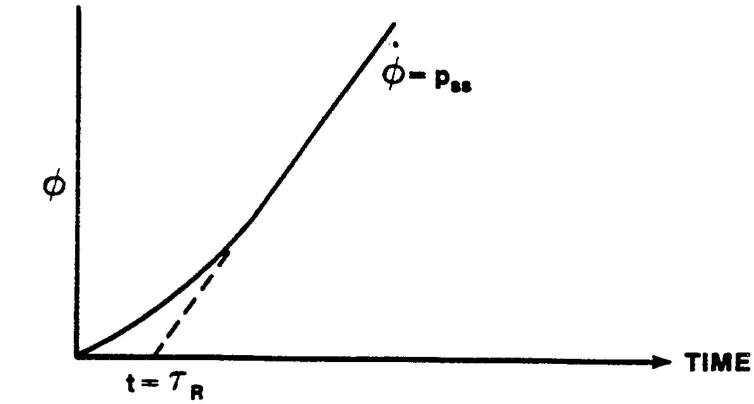


FIGURE 8.37A BANK ANGLE TRACE

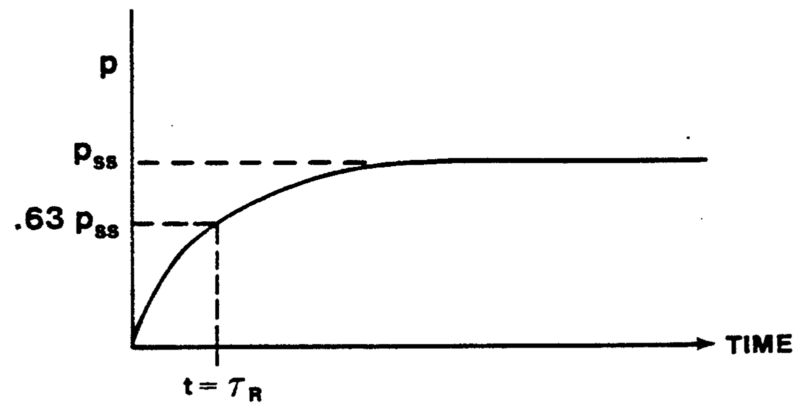


FIGURE 8.37B ROLL RATE TRACE

8.7.8 Roll-Sideslip Coupling

In contrast to most other requirements which specify desired response to control inputs, roll-sideslip coupling produces unwanted responses. The Dutch roll mode can be seen in the p and β traces. How these parameters are phased with each other will highlight closed loop problems. These unwanted responses detract from precise control and can contribute to PIO tendencies.

Roll-sideslip coupling is observed in at least three ways depending on the Dutch roll ϕ/β ratio.

Low ϕ/β Ratio (less than 1.5):

More sideslip than roll motion. In this case, if roll rate or aileron control excite sideslip, the flying qualities can be degraded by such motion

as an oscillation of the nose on the horizon during a turn or a lag or initial reversal in yaw rate during a turn entry or by pilot difficulty in quickly and precisely acquiring a given heading (ILS or GCA). In addition, the pilot has great difficulty damping Dutch roll with aileron only.

Large ϕ/β Ratio (1.5 to 6):

The coupling of β with p and ϕ becomes important, causing oscillations in roll rate and ratcheting of bank angle. Here, the pilot may have difficulty in precisely controlling roll rate or in acquiring a given bank angle without overshoots.

Very Large ϕ/β Ratio (>6):

Sensitivity of roll to rudder pedals or response to atmospheric disturbances may be so great that the aircraft response is never considered good.

In addition to the different problems caused by the magnitude of the ϕ/β ratio, the degree of difficulty in controlling these unwanted motions is very important. If the airplane is easy to coordinate during turn entries, then the pilot may tolerate relatively large unwanted motions during rudder pedal free turn entries since he can control these unwanted motions if desired. On the other hand, when coordination is difficult, the pilot will tolerate very small unwanted motions, since he must either accept these motions or may even aggravate them if he tries to coordinate. The parameter "ψ_β" was introduced as the most precise measure of this very nebulous, but important factor - difficulty of coordination. The use of "ψ_β" is primarily important when looking at small lateral control inputs and the resulting aircraft response in either roll or yaw.

8.7.8.1 Roll Rate Oscillations MIL- STD Requirements. Appendix A Paragraph 4.5.1.4 is primarily looking at aircraft with a Dutch roll ϕ/β ratio between 1.5 to 6.0 (moderate to large). This particular paragraph is specified for large inputs (at least 90° of roll). In general, any large oscillations after a step aileron input (rudder free) are not wanted. In the requirement

guidance section, the percentage values are given for different levels and categories which should not be exceeded. If there is a large change, the pilot will see ratcheting and won't like the aircraft's characteristics.

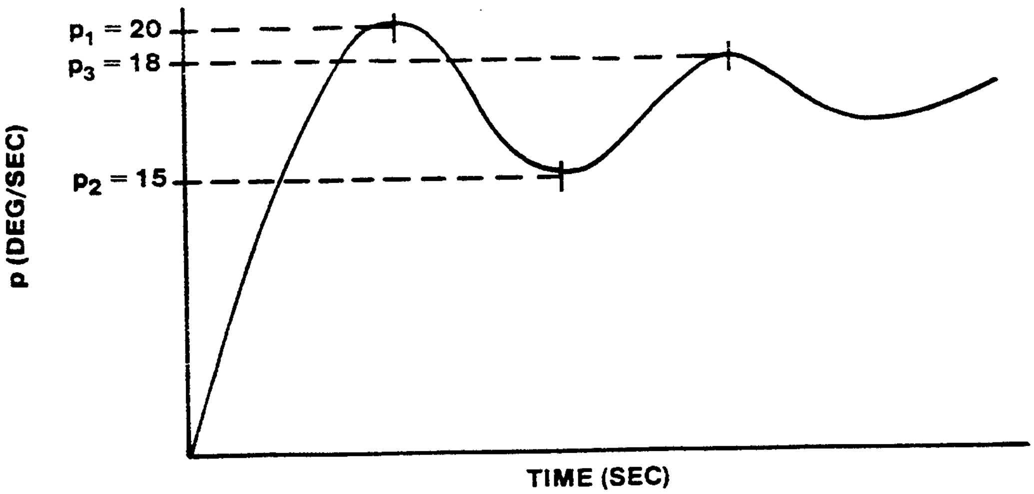


FIGURE 8.38 ROLL RATE OSCILLATIONS

p_1 is the first peak in the roll rate trace, and p_2 is the first minimum. The ratio of p_2/p_1 shall not exceed the values in the table. If p_2 crossed the time axis into negative territory, the sign of p has changed and an automatic failure should be given.

8.7.8.2 <u>Roll Rate Oscillation Limitations For Small Step Inputs</u>.

Appendix A Paragraph 4.5.1.4 addresses precision of control with small aileron inputs. The degree of oscillation - p_{osc}/p_{av} - are unwanted motions that pilots, who are flying an ILS or some other task, perceive as a performance difficulty. The larger the oscillation or the greater the difficulty in damping the oscillation, the greater the pilot's workload and ability to accomplish the task. In other words, the degradation in flying qualities is proportional to the amount of roll rate oscillation, p_{osc}, about some mean value of roll rate, p_{av}. The term "ψ_β" has been referred to as the "difficulty of coordination" parameter. This is based on the fact that an aircraft should develop adverse yaw with aileron inputs so that the pilot can normally coordinate the turn. For example, if a right roll is initiated with aileron, then the aircraft will yaw left (adverse), causing the pilot to use right

rudder to bring the adverse yaw to zero. Thus, he needs right rudder for right aileron inputs. If proverse yaw resulted from the input, then the pilot would have to cross control rudder and aileron to coordinate the roll. With this background, an examination of Figure 8.39 from Appendix A Paragraph 4.5.1.4 Figure 154 (page 393) can now be done.

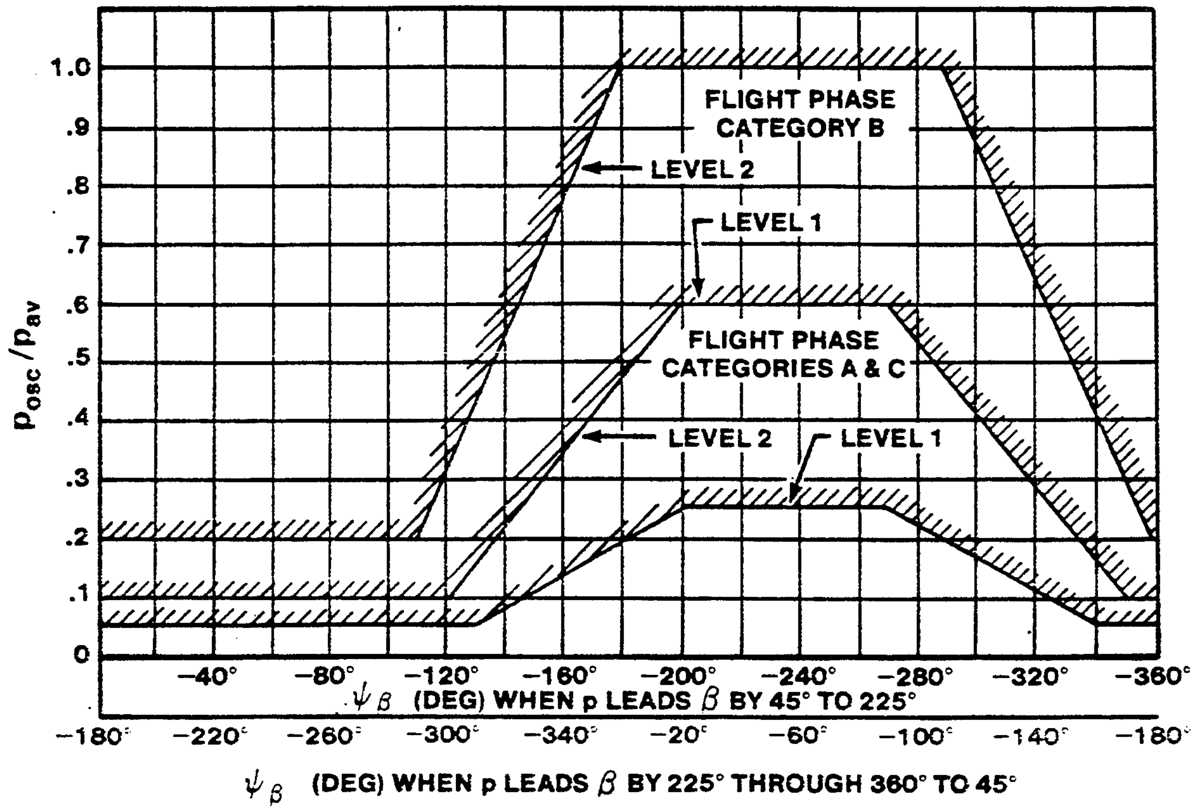


FIGURE 8.39 ROLL RATE OSCILLATION LIMITATIONS

From this figure it can be seen that the ratio of roll rate oscillation to steady state roll rate can be greater for some values of ψ_β than for others. The assumption that p leads β and that the aircraft has positive dihedral, will be made for the following discussion. Specifically, the specified values of p_{osc}/p_{av} for $0° \geq \psi_\beta \geq -90°$ are far more stringent than for $-180 \geq \psi_\beta \geq -270°$. There are at least three reasons for this.

First, aileron inputs proportional to bank angle errors generate yaw accelerations that tend to damp the Dutch roll oscillations when $-180° \geq \psi_\beta \geq -270°$. Thus, the Dutch roll oscillation damps out more quickly with pilot roll inputs. Conversely, if ψ_β is between 0° and −90°, the aileron input tends

to excite Dutch roll and can even cause lateral PIO. The latter case causes a pilot's tolerance of p_{osc}/p_{av} to be reduced.

Secondly, the requirements of p_{osc}/p_{av} vary considerably due to the difficulty of coordination previously mentioned. For $-180° \geqq \psi_\beta \geqq -270°$, normal coordination may be effected, that is, right rudder pedal is required for right rolls. Thus, even if roll oscillations do occur, the pilot can manage the oscillations by using rudders. For $0° \geqq \psi_\beta \geqq -90°$, it is necessary to cross control to effect coordination that is, left rudder for right aileron. Since pilots do not normally cross control, and if they must, they have great difficulty in doing so, they either let the oscillations go unchecked or make them worse.

The final reason for the significant variation in p_{osc}/p_{av} with ψ_β is that the average roll rate, p_{av}, for a given input varies significantly with ψ_β. For positive dihedral adverse yaw due-to-aileron ($\psi_\beta \approx 180°$) tends to decrease average roll rate, whereas proverse yaw-due-to-aileron ($\psi_\beta \approx 0°$) tends to increase roll rate. As a matter of fact, proverse yaw-due-to-aileron is sometimes referred to as "complementary yaw" because of this augmentation of roll effectiveness. Thus, for a given amplitude of p_{osc}, p_{osc}/p_{av} will be greater for $\psi_\beta \approx 180°$ than it will be at $\psi_\beta \approx 0°$.

p_{osc}/p_{av} and ψ_β are calculated using the following equations where p_{osc}/p_{av} depends on the value of ζ_{DR}.

$$\zeta_{DR} \leq .2 \qquad \frac{p_{osc}}{p_{av}} = \frac{p_1 + p_3 - 2p_2}{p_1 + p_3 + 2p_2}$$

$$\zeta_{DR} > .2 \qquad \frac{p_{osc}}{p_{av}} = \frac{p_1 - p_2}{p + p_2}$$

where p_1, p_2, and p_3 are roll rates at the first, second and third peaks respectively. See Figure 8.40. In order to calculate ψ_β, a sign convention must be assumed. First, assume positive dihedral aircraft (p leading β will verify this). Second, use upper set of ψ_β numbers in all paragraphs showing

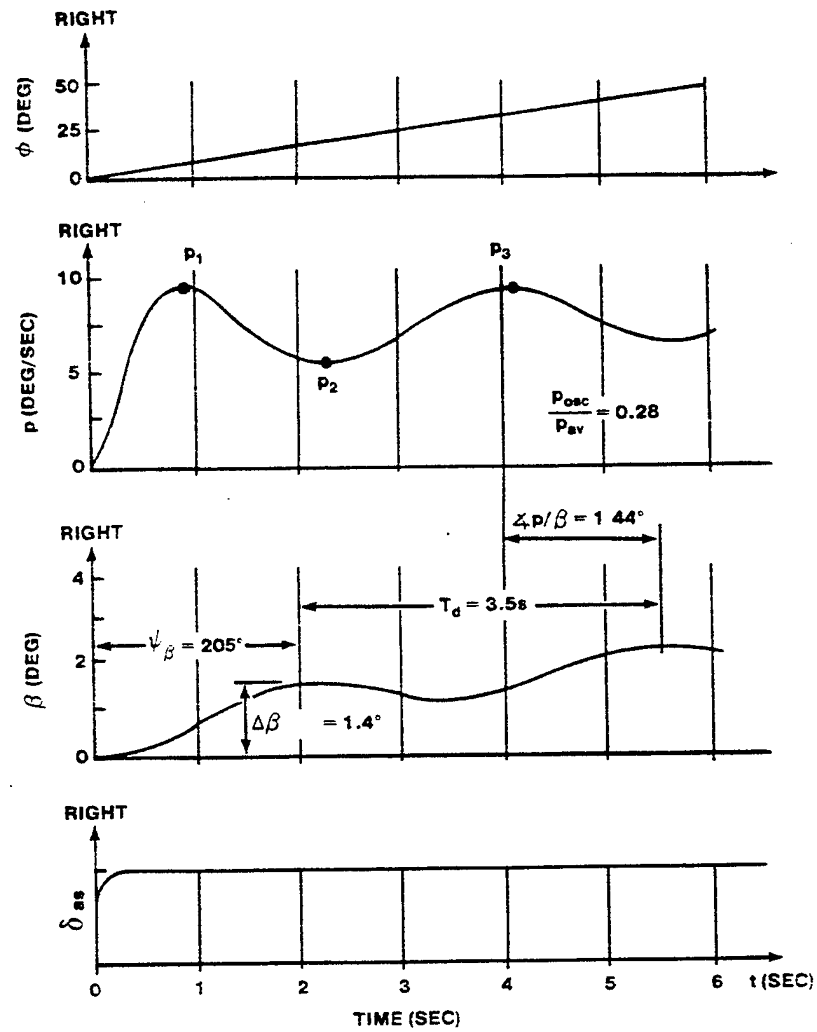


FIGURE 8.40 ROLL RATE OSCILLATION DETERMINATION

two sets. (This can be verified by calculating the angle between the p and β trace maximums.) Third, if the roll is made to the right, look for the 1st local maximum on the β trace, and if the roll is made to the left, look for the 1st local minimum. Finally, use the formula

$$\psi_\beta = -\frac{360}{T_d} t_{n_\beta} + (n - 1)\ 360 \text{ deg}$$

n = nth Local Maximum

T_D is by definition the Dutch roll period (shown in Figure 8.40); t_{n_β} is the time on the β trace for the 1st local maximum (right roll) or 1st local minimum (left roll). See Figure 8.40 for a right roll. If $\measuredangle$ p/β is calculated, just compare the time difference between the p and β trace and divide by the T_d and multiply by 360°.

The only other requirement is to make sure that the input is small. It should take at least 1.7 times T_d for a 60° bank angle change.

8.7.8.3 Bank Angle Oscillation MIL- STD Limitations

In order to extend the roll-sideslip coupling requirement to larger control deflections and to account for some flight control non-linearities, Appendix A Paragraph 4.5.1.4 specifies similar calculations as for roll rate limitations except that the input is an impulse. This input should be at the maximum rate and at the largest deflection possible. The resulting motion after the input will be bank angle oscillations around zero degrees. The aileron impulse should be applied after being stabilized at approximately 15° of bank. The difference in the shifting of curve in Figure 8.41B (Appendix A Figure 155, page 393) is due to ψ_β from a pulse being 90° more positive than for a step.

To calculate ϕ_{osc}/ϕ_{av} identify bank angles ϕ_1, ϕ_2, and ϕ_3 as shown on Figure 8.41A.

For $\zeta_{DR} \leqq .2$ $$\frac{\phi_{osc}}{\phi_{av}} = \frac{\phi_1 + \phi_3 - 2\phi_2}{\phi_1 + \phi_2 + 2\phi_2}$$

For $\zeta_{DR} > .2$ $$\frac{\phi_{osc}}{\phi_{av}} = \frac{\phi_1 - \phi_2}{\phi_1 + \phi_2}$$

Next calculate ψ_β as was discussed previously on the β trace.

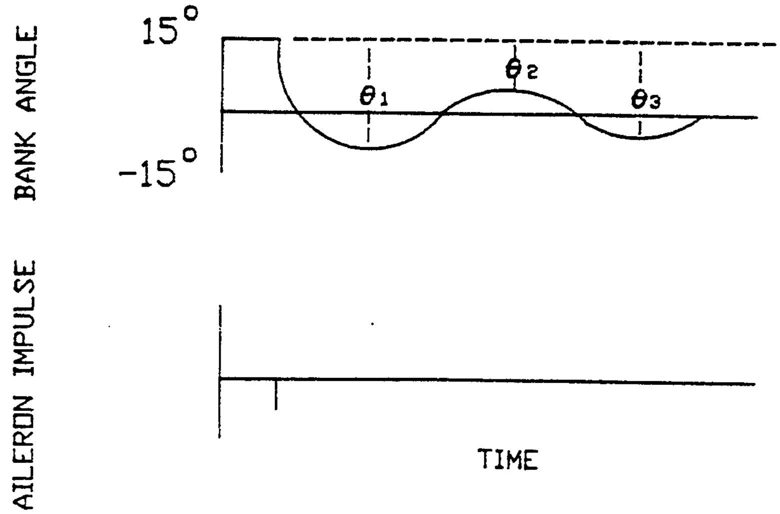


FIGURE 8.41A BANK ANGLE OSCILLATIONS

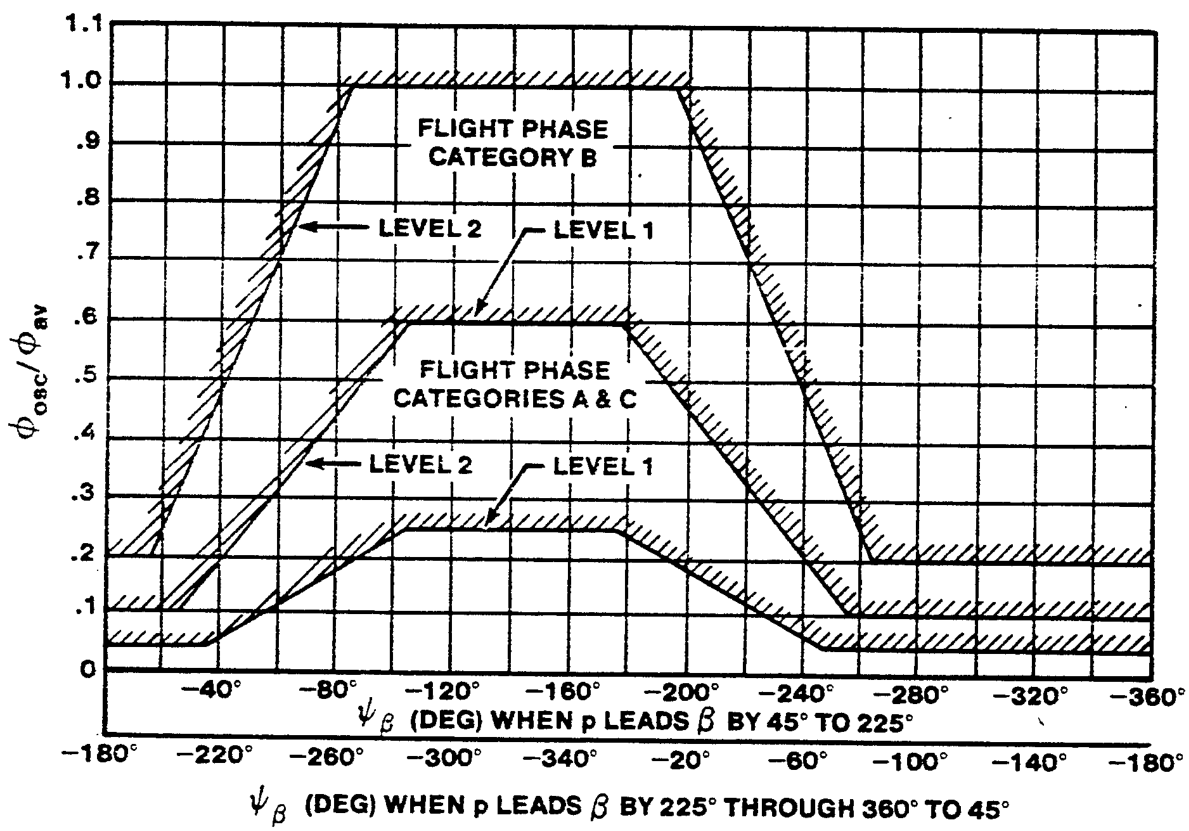


FIGURE 8.41B BANK ANGLE OSCILLATION LIMITATIONS

8.7.9 Yaw Axis Response to Roll Control

Appendix A Paragraph 4.6.2 and the next one for small inputs are for low Dutch roll ($\phi/\beta < 1.5$) and are associated with sideslip rather than roll or bank angle tracking. The basis for the paragraph is research indicating a maximum amount of sideslip generated that can be tolerated by the pilot, whether the sideslip is adverse or proverse and the phase relationship between the sideslip and the roll in the Dutch roll are the overriding factors. The coordination of control with proverse yaw is very difficult and unnatural so the levels specified are much lower than those for adverse yaw. Another factor to be considered is the side force and side acceleration caused by sideslip angles at high speed. Research has concluded that if such acceleration is very high (> .2 g's), then the resulting motions cause interference with normal pilot duties.

In order to calculate the required parameters for this paragraph, refer to Figure 8.40 for $\Delta\beta$ and k. The Dutch roll period is determined as before, and half its value is compared with two seconds. Whichever is greater, the β excursion proverse yaw and adverse yaw needs to be used during that time after the rudder pedal free aileron input. From Figure 8.40 there is no proverse yaw, but 1.4° of adverse yaw during two seconds of right roll is present. Many tests will have a small proverse yaw excursion before adverse yaw builds up, which would require a calculation of $\Delta\beta$ proverse. Next, calculate the "k" factor. It is known as the "severity of input" parameter. "k" is the ratio of what roll rate was achieved during the flight test, (ϕ_t) command, versus what roll rate is required (ϕ_t) by 4.5.8.1 roll paragraphs for the particular aircraft Class and Flight Phase.

$$k = \frac{(\phi_t) \text{ commanded (what you did)}}{(\phi_t) \text{ required (MIL STD Req)}}$$

Next, calculate $\Delta\beta/k$ proverse and $\Delta\beta/k$ adverse and compare them to the requirements of the paragraph. This test must occur with the aileron step fixed for at least 90° of bank angle change.

8.7.9.1 Yaw Axis Response to Roll Control Requirement For Small Inputs. The requirements for are similar to those seen in the roll rate paragraph for small inputs. Even though the roll paragraph applied to

problems with roll as opposed to sideslip, the pilot opinions were similar when coordination is required with adverse or proverse yaw. The difference in the sketch of this paragraph is almost totally due to the difference in ability to coordinate during turn entries and exits. As ψ_β varies from 0° to -360°, it indicates the coordination problem discussed previously. When adverse yaw is present $-180° \geqq \psi_\beta \geqq -270°$, coordination is easy and oscillations can be readily minimized. As more proverse yaw is seen $-360° \geqq \psi_\beta \geqq -90°$, cross controlling is required and the oscillations go unchecked or are amplified by pilot's efforts to coordinate with rudder pedal.

Only one new parameter needs to be calculated for this paragraph - $\Delta\beta$. It is the total algebraic change of β during half the Dutch roll period or two seconds, whichever is greater. Next calculate k and ψ_β as discussed before. Make sure that the sign convention for ψ_β remains the same (Figure 8.42). The only other requirement is that dictated for the size of the input. It should be small enough that a 60° bank angle change takes more than the Dutch roll period or two seconds, whichever is longer.

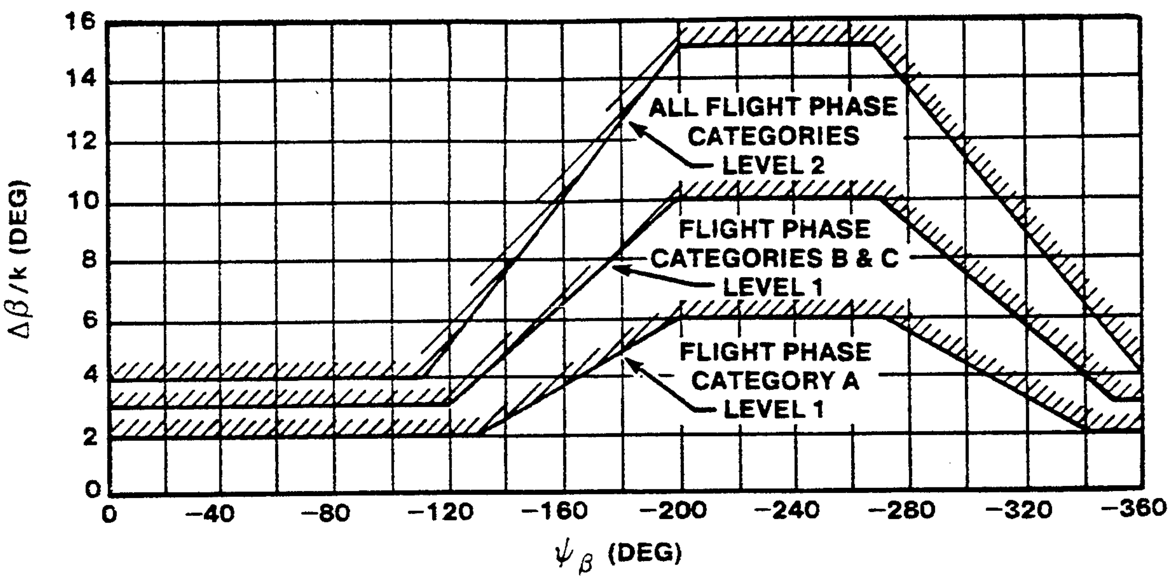


FIGURE 8.42. SIDESLIP EXCURSIONS FOR SMALL INPUTS

8.8 SUMMARY

The dynamic stability flight test methods discussed in this chapter can be used to investigate the five major modes of an aircraft's free response motion. These dynamic stability and control investigations, when coupled with pilot in the loop task analysis, will determine the acceptability of an aircraft's dynamic response characteristics.

8.9 HANDLING QUALITIES

Because the "goodness" with which an aircraft flies is often stated as a general appraisal . . . "My F-69 is the best damn fighter ever built, and it can outfly and outshoot any other airplane." "It flies good." "That was really hairy." . . . you probably can understand the difficulty of measuring how well an aircraft handles. The basic question of what parameters to measure and how those parameters relate to good handling qualities has been a difficult one, and the total answer is not yet available. The current best answers for military aircraft are found in MIL-STD-1797, the specification for the "Flying Qualities of Piloted Airplanes."

When an aircraft is designed for performance, the design team has definite goals to work toward . . . a particular takeoff distance, a minimum time to climb, or a specified combat radius. If an aircraft is also to be designed to handle well, it is necessary to have some definite handling quality goals to work toward. Success in attaining these goals can be measured by flight tests for handling qualities when some rather firm standards are available against which to measure and from which to recommend.

In order to make it possible to specify acceptable handling qualities, it was necessary to evolve some flight test measurable parameters. Flight testing results in data which yield values for the various handling quality parameters, and the military specification gives a range of values that should ensure good handling qualities. Because MIL-STD-1797 is not the ultimate answer, the role of the test pilot in making accurate qualitative observations and reports in addition to generating the quantitative data is of great importance in handling qualities testing.

One method that has been extensively used in handling qualities quantification is the use of pilot opinion surveys and variable stability aircraft. For example, a best range of values for the short period damping ratio and natural frequency could be identified by flying a particular aircraft type to accomplish a specific task while allowing the ζ and ω_n to vary. From the opinions of a large number of pilots, a valid best range of values for ζ and ω_n could be obtained, as shown in Figure 8.43.

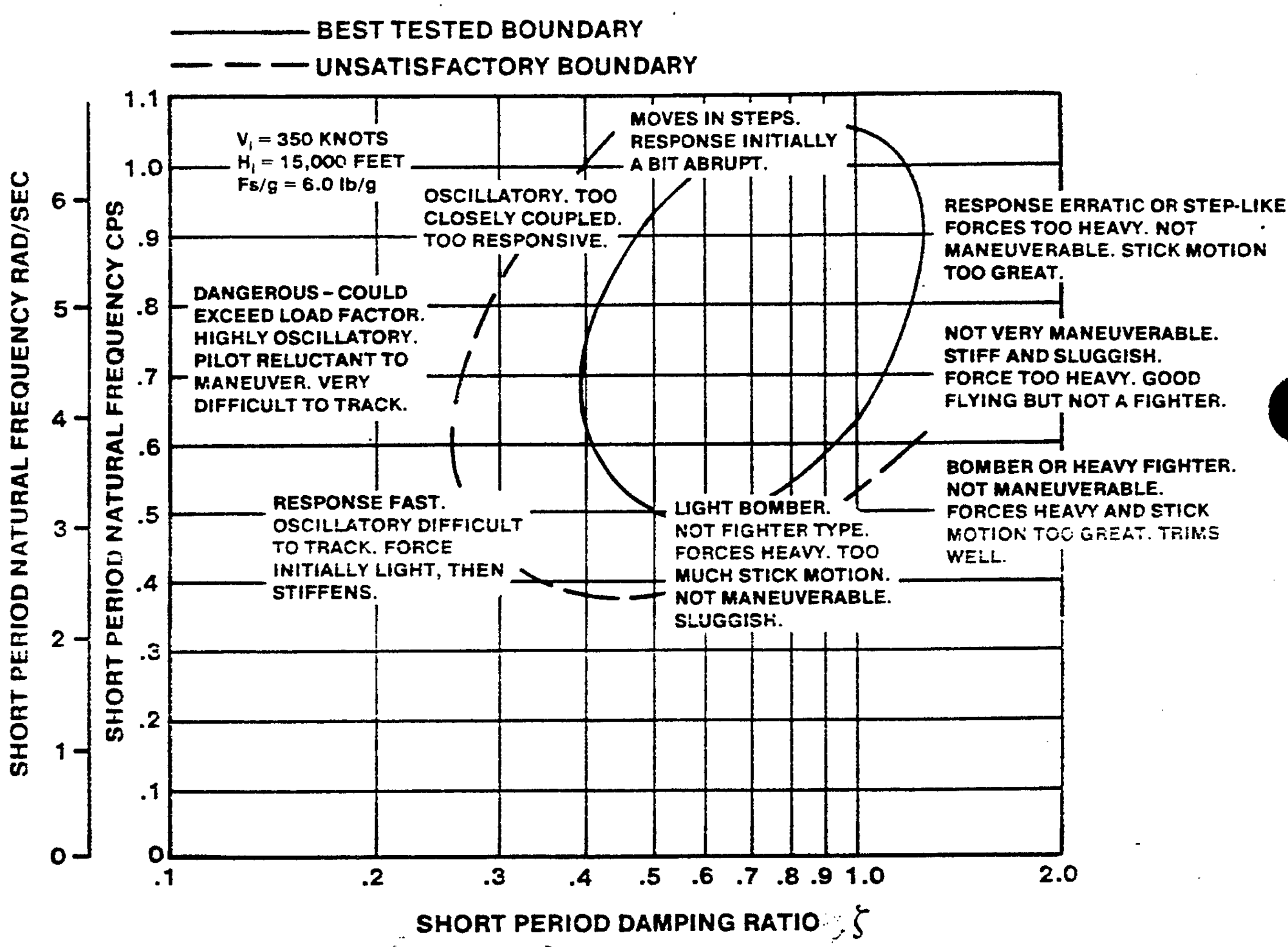


FIGURE 8.43 BEST RANGE FOR ζ AND ω_n FROM PILOT OPINION

8.9.1 Open Loop Vs Closed Loop Response

The ζ and ω_n being discussed here are the aircraft free or open loop response characteristics which describe aircraft motion without pilot inputs. With the pilot in the loop, the free response of the aircraft is hidden as pilot inputs are continually made. The closed loop block diagram shown in Figure 8.44 can be used to understand aircraft closed loop response.

The free response of an aircraft relates directly to how well the aircraft can be flown with a pilot in the loop, and many of the pertinent handling qualities parameters are for the open loop aircraft.

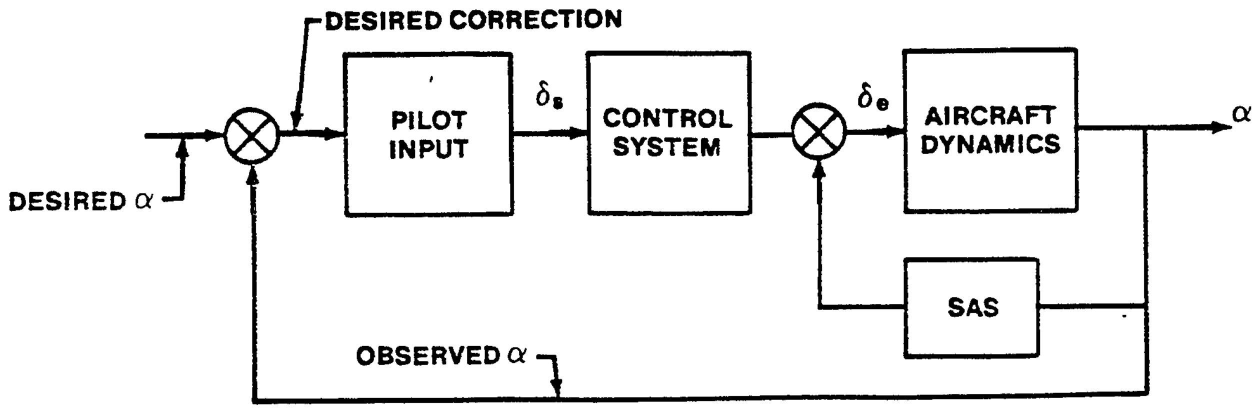


FIGURE 8.44 CLOSED LOOP BLOCK DIAGRAM

The real test of an aircraft's handling qualities is how well it can be flown closed loop to accomplish a particular mission. Closed loop handling quality evaluations such as air-to-air tracking in simulated air combat maneuvering play an important part of determining how well an aircraft handles.

8.9.2 Pilot In The Loop Dynamic Analysis

Calspan (formerly Cornell Aeronautical Laboratory) has made notable contributions to the understanding of pilot rating scales and pilot opinion surveys. Except for minor variations between pilots, which sometimes prevent a sharp delineation between acceptable and unacceptable flight characteristics, there is very definite consistency and reliability in pilot opinion. In addition, the opinions of well qualified test pilots can be exploited because of their engineering knowledge and experience in many different aircraft types.

The stability and control characteristics of airplanes are generally established by wind tunnel measurement and by technical analysis as part of the airplane design process. The handling qualities of a particular airplane are related to the stability and control characteristics. The relationship is a complex one which involves the combination of the airplane and its pilot in the accomplishment of the intended mission. It is important that the effects of specific stability and control characteristics be evaluated in terms of their ultimate effects on the suitability of the pilot-vehicle combination for

the mission. On the basis of this information, intelligent decisions can be made during the airplane design phase which will lead to the desired handling qualities of the final product.

There are three general ways in which the relationship between stability and control parameters and the degree of suitability of the airplane for the mission may be examined:

1. Theoretical analysis
2. Experimental performance measurement
3. Pilot evaluation

Each of the three approaches has an important role in the complete evaluation. One might ask, however, why is the pilot assessment necessary? At present a mathematical representation of the human operator best lends itself to analysis of specific simple tasks. Since the intended use is made up of several tasks and several modes of pilot-vehicle behavior, difficulty is experienced first in accurately describing all modes analytically, and second in integrating the quality of the subordinate parts into a measure of overall quality for the intended use. In spite of these difficulties, theoretical analysis is fundamental for understanding pilot-vehicle difficulties, and pilot evaluation without it remains a purely experimental process.

Attaining satisfactory performance in a designated mission is a fundamental reason for our concern with handling qualities. Why can't the experimental measurement of performance replace pilot evaluation? Why not measure pilot-vehicle performance in the intended use - isn't good performance consonant with good quality? A significant difficulty arises here in that the performance measurement tasks may not demand of the pilot all that the real mission demands. The pilot is an adaptive controller whose goal is to achieve good performance. In a specific task, he is capable of attaining essentially the same performance for a wide range of vehicle characteristics, at the expense of significant reductions in his capacity to assume other duties and planning operations. Significant differences in task performance may not be measured where very real differences in mission suitability do exist.

The questions which arise in using performance measurements may be summarized as follows: (1) For what maneuvers and tasks should measurements be made to define the mission suitability? (2) How do we integrate and weigh the performance in several tasks to give an overall measure of quality if measurable differences do exist? (3) Is it necessary to measure or evaluate pilot workload and attention factors for performance to be meaningful? If so, how are these factors weighed with those in (2)? (4) What disturbances and distractions are necessary to provide a realistic workload for the pilot during the measurement of his performance in the specified task?

Pilot evaluation still remains the only method of assessing the interactions between pilot performance and workload in determining suitability of the airplane for the mission. It is required in order to provide a basic measure of quality and to serve as a standard against which pilot-airplane system theory may be developed, against which performance measurements may be correlated, and with which significant airplane design parameters may be determined and correlated.

The technical content of the pilot evaluation generally falls into two categories: one, the identification of characteristics which interfere with the intended use, and two, the determination of the extent to which these characteristics affect mission accomplishment. The latter judgment may be formalized as a pilot rating.

8.9.3 Pilot Rating Scales

In 1956, the newly formed Society of Experimental Test Pilots accepted responsibility for one program session at the annual meeting of the Institute of Aeronautical Sciences. A paper entitled "Understanding and Interpreting Pilot Opinion" was presented with the intent to create better understanding and use of pilot opinion in aeronautical research and development. The widespread use of rating systems has indicated a general need for some uniform method of assessing aircraft handling qualities through pilot opinion.

Several rating scales were independently developed during the early use of variable stability aircraft. These vehicles, as well as the use of ground simulation, made possible systematic studies of aircraft handling qualities through pilot evaluation and rating of the effects of specific stability and control parameters.

Figure 8.45 shows the 10-point Cooper-Harper Rating Scale that is widely used today.

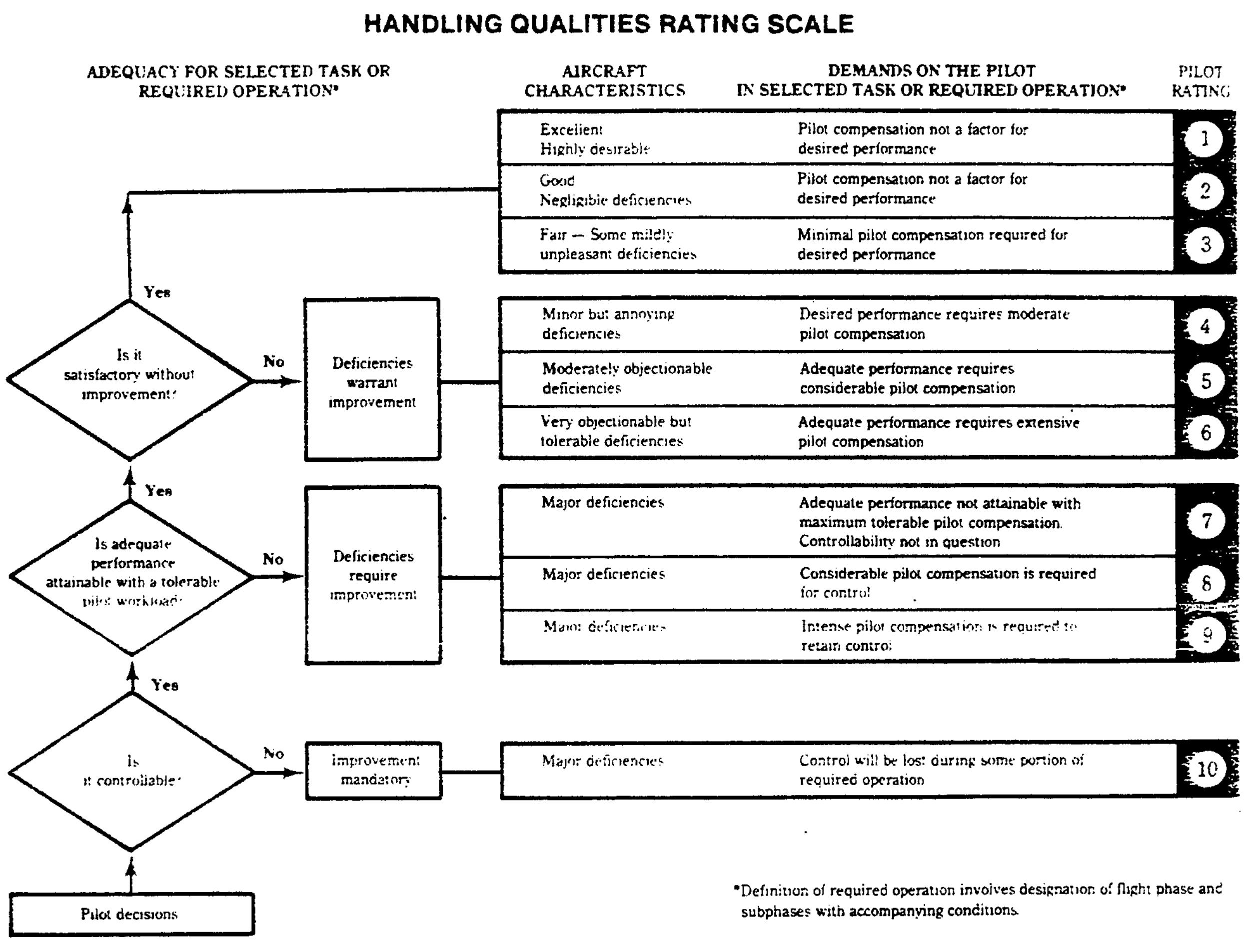


FIGURE 8.45 TEN-POINT COOPER-HARPER PILOT RATING SCALE

A flow chart is shown in Figure 8.46 that traces the series of dichotomous decisions that the pilot makes in arriving at the final rating. As a rule, the first decision may be fairly obvious. Is the configuration controllable or uncontrollable? Subsequent decisions become less obvious as the final rating is approached.

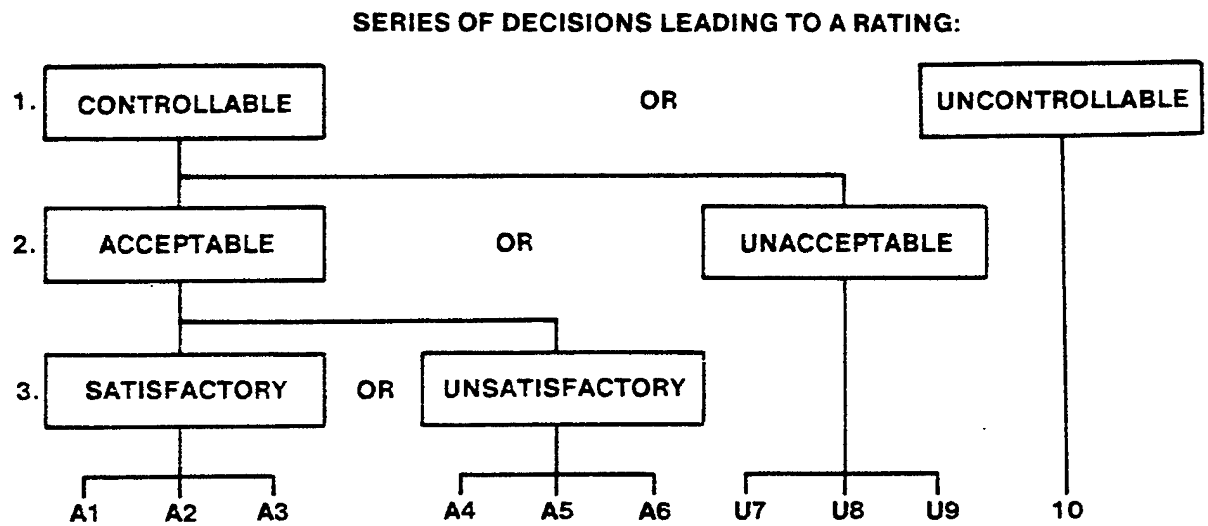


FIGURE 8.46 SEQUENTIAL PILOT RATING DECISIONS

If the airplane is uncontrollable in the mission, it is rated 10. If it is controllable, the second decision examines whether it is acceptable or unacceptable. If unacceptable, the ratings U7, U8, and U9 are considered (rating 10 has been excluded by the "controllable" answer to the first decision). If it is acceptable, the third decision must examine whether it is satisfactory or unsatisfactory. If unsatisfactory, the ratings 4, 5, and 6 are considered; if satisfactory, the ratings 1, 2, and 3 are considered.

The basic categories must be described in carefully selected terms to clarify and standardize the boundaries desired. Following a careful review of dictionary definitions and consideration of the pilot's requirement for clear, concise descriptions, the category definitions shown in Figure 8.26 were selected. When considered in conjunction with the structural outline presented in Figure 8.46 a clearer picture is obtained of the series of decisions which the pilot must make.

CATEGORY	DEFINITION
CONTROLLABLE	CAPABLE OF BEING CONTROLLED OR MANAGED IN CONTEXT OF MISSION, WITH AVAILABLE PILOT ATTENTION.
UNCONTROLLABLE	CONTROL WILL BE LOST DURING SOME PORTION OF MISSION.
ACCEPTABLE	MAY HAVE DEFICIENCIES WHICH WARRANT IMPROVEMENT BUT ADEQUATE FOR MISSION. PILOT COMPENSATION, IF REQUIRED TO ACHIEVE ACCEPTABLE PERFORMANCE, IS FEASIBLE.
UNACCEPTABLE	DEFICIENCIES WHICH REQUIRE MANDATORY IMPROVEMENT. INADEQUATE PERFORMANCE FOR MISSION, EVEN WITH MAXIMUM FEASIBLE PILOT COMPENSATION.
SATISFACTORY	MEETS ALL REQUIREMENTS AND EXPECTATIONS; GOOD ENOUGH WITHOUT IMPROVEMENT. CLEARLY ADEQUATE FOR MISSION.
UNSATISFACTORY	RELUCTANTLY ACCEPTABLE. DEFICIENCIES WHICH WARRANT IMPROVEMENT. PERFORMANCE ADEQUATE FOR MISSION WITH FEASIBLE PILOT COMPENSATION.

FIGURE 8.47 MAJOR CATEGORY DEFINITIONS

8.9.4 Major Category Definitions

To control is to exercise direction of, or to command. Control also means to regulate. The determination as to whether the airplane is controllable or not must be made within the framework of the defined mission or intended use. An example of the considerations of this decision would be the evaluation of fighter handling qualities during which the evaluation pilot encounters a configuration over which he can maintain control only with his complete and undivided attention. The configuration is "controllable" in the sense that the pilot can maintain control by restricting the tasks and maneuvers which he is called upon to perform and by giving the configuration his undivided attention. However, for him to answer "Yes, it is controllable in the mission," he must be able to retain control in the mission tasks with whatever effort and attention are available from the totality of his mission duties. Uncontrollable implies that flight manual limitations may be exceeded during performance of the mission task.

The dictionary shows that "acceptable" means that a thing offered is received with a consenting mind; "unacceptable" means that it is refused or rejected. Acceptable means that the mission can be accomplished;

it means that the evaluation pilot would agree to buy it for the mission to fly, for his son to fly, or for either to ride in as a passenger. "Acceptable" in the rating scale doesn't say how good it is for the mission, but it does say it is good enough. With these characteristics, the mission can be accomplished. It may be accomplished with considerable expenditure of effort and concentration on the part of the pilot, but the levels of effort and concentration required in order to achieve this acceptable performance are feasible in the intended use. By the same token, unacceptable does not necessarily mean that the mission cannot be accomplished; it does mean that the effort, concentration, and workload necessary to accomplish the tasks are of such a magnitude that the evaluation pilot rejects that airplane for the mission.

Consider now a definition of satisfactory. The dictionary defines this as adequate for the purpose. A pilot's definition of satisfactory might be that it isn't necessarily perfect or even good, but it is good enough that he wouldn't ask that it be fixed. It meets a standard, it has sufficient goodness and it can meet all requirements of a mission task. Acceptable but unsatisfactory implies that it is acceptable even though objectionable characteristics should be improved, that it is deficient in a limited sense, or that there is insufficient goodness. Thus, the quality is either:

1. Acceptable (satisfactory) and therefore of the best category,
   or
2. Acceptable (unsatisfactory) and of the next best category,
   or
3. Unacceptable. Not suitable for the mission, but still controllable,
   or
4. Unacceptable for the mission and uncontrollable.

8.9.5 Experimental Use Of Rating Of Handling Qualities

The evaluation of handling qualities has a similarity to other scientific experiments in that the output data are only as good as the care taken in the design and execution of the experiment itself and in the analysis and reporting of the results. There are two basic categories of output data in a handling qualities evaluation: the pilot comment data and the pilot ratings. Both items are important output data. An experiment which ignores one of the two outputs is discarding a substantial part of the output information.

The output data which are most often neglected are the pilot comments, primarily because they are quite difficult to deal with due to their qualitative form and, perhaps their bulk. Ratings, however, without the attendant pilot objections, are only part of the story. Only if the deficient areas can be identified can one expect to devise improvements to eliminate or attenuate the shortcomings. The pilot comments are the means by which the identification can be made.

8.9.6 Mission Definition

Explicit definition of the mission (task) is probably the most important contributor to the objectivity of the pilot evaluation data. The mission (task) is defined here as a use to which the pilot-airplane combination is to be put. The mission must be very carefully examined, and a clear definition and understanding must be reached between the engineer and the evaluation pilot as to their interpretation of this mission. This definition must include:

1. What the pilot is required to accomplish with the airplane
2. The conditions or circumstances under which he must perform the task

For example, the conditions or circumstances might include instrument or visual flight or both, type of displays in the cockpit, input information to assist the pilot in the accomplishment of the task, etc. The environment in which the task is to be accomplished must also be defined and considered in the evaluation, and could include, for example, the presence or absence of turbulence, day versus night, the frequency with which the task has to be repeated, the variability in pilot preparedness for the task and his proficiency level.

8.9.7 Simulation Situation

The pilot evaluation is seldom conducted under the circumstances of the real mission. The evaluation almost inherently involves simulation to some degree because of the absence of the real situation. As an example, the

evaluation of a day fighter is seldom carried out under the circumstances of a combat mission in which the pilot is not only shooting at real targets, but also being shot back at by real guns. Therefore, after the mission has been defined, the relationship of the simulation situation to the real mission must be explicitly stated for both the engineer and the evaluation pilot so that each may clearly understand the limitations of the simulation situation.

The pilot and engineer must both know what is left out of the evaluation program, and also what is included that should not be. The fact that the anxiety and tension of the real situation are missing, and that the airplane is flying in the clear blue of calm daylight air, instead of in the icing, cloudy, turbulent, dark situation of the real mission, will affect results. Regardless of the evaluation tasks selected, the pilot must use his knowledge and experience to provide a rating which includes all considerations which are pertinent to the mission, whether provided in the tasks or not.

8.9.8 Pilot Comment Data

One of the fallacies resulting from the use of a rating scale which is considered for universal handling qualities application is the assumption that the numerical pilot rating can represent the entire qualitative assessment. Extreme care must be taken against this oversimplification because it does not constitute the full data gathering process.

Pilot objections to the handling qualities are important, particularly to the airplane designer who is responsible for the improvement of the handling qualities. But, even more important, the pilot comment data are essential to the engineer who is attempting to understand and use the pilot rating data. If ratings are the only output data, one has no real way of assessing whether the objectives of the experiment were actually realized. Pilot comments supply a means of assessing whether the pilot objections (which lead to his summary rating) were related to the mission or resulted from some extraneous uncontrolled factor in the execution of the experiment, or from individual pilots focusing on and weighing differently various aspects of the mission. Attention to detail is important to ensure that pilot comments are useful.

Pilots must comment in the simplest language. Avoid engineering terms unless they are carefully defined. The pilot should report what he sees and

feels, and describe his difficulties in carrying out that which he is attempting. It is then important for the pilot to relate the difficulties which he is having in executing specific tasks to their effect on the accomplishment of the mission.

The pilot should make specific comments in evaluating each configuration. These comments generally are in response to questions which have been developed in the discussions of the mission and simulation situation. The pilot must be free to comment on difficulties over and above the specific questions asked of him. The test pilot should strive for a balance between a continuous running commentary and occasional comment in the form of an explicit adjective. The former often requires so much editing to find the substance that it is often ignored, while the latter may add nothing to the numerical rating itself.

The pilot comments must be taken during or immediately after each evalution. In-flight comments should be recorded on a tape recorder. Experience has shown that the best free comments are often given during the evaluation. If the comments are left until the conclusion of the evaluation, they are often forgotten. A useful procedure is to permit free comment during the evaluation itself and to require answers to specific questions in the summary comments at the end of the evaluation.

Questionnaires and supplementary pilot comments are most necessary to ensure that: (a) all important or suspected aspects are considered and not overlooked, (b) information is provided relative to why a given rating has been given, (c) an understanding is provided of the tradeoffs with which pilots must continually contend, and (d) supplementary comment that might not be offered otherwise is stimulated. It is recommended that the pilots participate in the preparation of the questionnaires. The questionnaires should be modified if necessary as a result of the pilots' initial evaluations.

8.9.9 Pilot Rating Data

The pilot rating is an overall summation of the pilot observations relating to the mission. The basic question that is asked of the pilot conditions the answer that he provides. For this reason, it is

important that the program objectives are clearly stated and understood by all concerned and that all criteria, whether established or assumed, be clearly defined. In other words, it is extremely important that the basis upon which the evaluation is established be firmly understood by pilots and engineers. Unless a common basis is used, one cannot hope to achieve comparable pilot ratings, and confusing disagreement will often result. Care must also be taken that criteria established at the beginning of the program carry through to the end. If the pilot finds it necessary to modify his tasks, technique or mission definition during the program, he must make it clear just when this change occurred.

A discussion of the specific use of a rating scale tends to indicate some disagreement among pilots as to how they actually arrive at a specific numerical rating. There is general agreement that the numerical rating is only a shorthand for the word definition. Some pilots, however, lean heavily on the specific adjective description and look for that description which best fits their overall assessment. Other pilots prefer to make the decisions sequentially, thereby arriving at a choice between two or three ratings. The decision among the two or three ratings is then based upon the adjective description. In concept, the latter technique is preferred since it emphasizes the relationship of all decisions to the mission.

The actual technique used is somewhere between the two techniques above and not so different among pilots. In the past, the pilot's choice has probably been strongly influenced by the relative usefulness of the descriptions provided for the categories on the one hand, and the numerical ratings on the other. The evaluation pilot is continuously considering the rating decision process during his evaluation. He proceeds through the decisions to the adjective descriptors enough times that his final decision is a blend of both techniques. It is therefore obvious that descriptors should not be contradictory to the mission-oriented framework.

Half ratings are permitted (e.g., rating 4.5) and are generally used by the evaluation pilot to indicate a reluctance to assign either of the adjacent ratings to describe the configuration. Any finer breakdown than half ratings is prohibited since any number greater than or less than the half rating implies that it belongs in the adjacent group. Any distinction between

configurations assigned the same rating must be made in the pilot comments. Use of the 3.5, 6.5, and 9.5 ratings is discouraged as they must be interpreted as evidence that the pilot is unable to make the fundamental decision with respect to category.

As noted previously, the pilot rating and comments must be given on the spot in order to be most meaningful. If the pilot should later want to change his rating, the engineer should record the reasons and the new rating for consideration in the analysis, and should attempt to repeat the configuration later in the evaluation program. If the configuration cannot be repeated, the larger weight (in most circumstances) should be given to the on-the-spot rating since it was given when all the characteristics were fresh in the pilot's mind.

8.9.10 Execution Of Handling Qualities Tests

Probably the most important item is the admonition to execute the test as it was planned. It is valuable for the engineer to monitor the pilot comment data as the test is conducted in order that he becomes aware of evaluation difficulties as soon as they occur. These difficulties may take a variety of forms. The pilot may use words which the engineer needs to have defined. The pilot's word descriptions may not convey a clear, understandable picture of the piloting difficulties. Direct communication between pilot and engineer is most important in clarifying such uncertainties. In fact, communication is probably the most important single element in the evaluation of handling qualities. Pilot and engineer must endeavor to understand one another and cooperate to achieve and retain this understanding. The very nature of the experiment itself makes this somewhat difficult. The engineer is usually not present during the evaluation, and hence he has only the pilot's word description of any piloting difficulty. Often, these described difficulties are contrary to the intuitive judgments of the engineer based on the characteristics of the airplane by itself. Mutual confidence is required. The engineer should be confident that the pilot will give him accurate,meaningful data; the pilot should be confident that the engineer is vitally interested in what he has to say and trusts the accuracy of his comments.

It is important that the pilot have no foreknowledge of the specific characteristics of the configuration being investigated. This does not exclude information which can be provided to help shorten certain tests (e.g., the parameter variations are lateral-directional, only). But it does exclude foreknowledge of the specific parameters under evaluation. The pilot must be free to examine the configuration without prejudice, learn all he can about it from meeting it as an unknown for the first time, look clearly and accurately at his difficulties in performing the evaluation task, and freely associate these difficulties with their effects on the ultimate success of the mission. A considerable aid to the pilot in this assessment is to present the configuration in a random fashion.

The amount of time which the pilot should use for the evaluation is difficult to specify a priori. He is normally asked to examine each configuration for as long as is necessary to feel confident that he can give a reliable and repeatable assessment. Sometimes, however, it is necessary to limit the evaluation time to a specific period of time because of circumstances beyond the control of the researcher. If the evaluation time per pilot is limited, a larger sample of pilots, or repeat evaluations will be required for similar accuracy, and the pilot comment data will be of poorer quality.

The evaluation pilot must be confident of the importance of the simulation program and join wholeheartedly into the production of data which will supply answers to the questions. Pilots as a group are strongly motivated toward the production of data to improve the handling qualities of the airplanes they fly. It isn't usually necessary to explicitly motivate the pilot, but it is very important to inspire his confidence in the structure of the experiment and the usefulness of his rating and comment data. Pilot evaluations are probably one of the most difficult tasks that a pilot undertakes. To produce useful data involves a lot of hard work, tenacity, and careful thought. There is a strong tendency for the pilot to become discouraged about their ultimate usefulness. The pilot needs feedback on the accuracy and repeatability of his evaluations. The test pilot is the only one who can provide the answers to the questions that are being asked. He must be reassured through feedback that his assessments are good so that he gains confidence in the manner in which he is carrying out the program.

PROBLEMS

8.1. Define Static Stability

8.2. Define:

a. Positive Static Stability
b. Neutral Static Stability
c. Negative Static Stability

8.3. Define Dynamic Stability

8.4. Define:

a. Positive Dynamic Stability
b. Neutral Dynamic Stability
c. Negative Dynamic Stability

8.5. List two assumptions for using the LaPlace Transformation to find the characteristic equation of a given system.

8.6. Given the following expression, determine the steady state roll rate due to an aileron step input of 10°.

$$.3\phi(t) + .5\dot{\phi}(t) = .7\delta_a(t) \qquad\qquad p(t) \simeq \dot{\phi}(t)$$

8.7. For Problem 8.6, what is the time constant (τ).

8.8. An aircraft is in the design stage and the following set of equations predict one of the modes of motion about the longitudinal axis (short period):

$$13.78\ \dot{\alpha}(t) + 4.5\ \alpha(t) - 13.78\ \theta(t) = -.25\ \delta_e(t)$$

$$.055\ \dot{\alpha}(t) + .619\ \alpha(t) + .514\ \theta(t) + .19\ \theta(t) = -.71\ \delta_e(t)$$

a. Determine the characteristic equation.

b. Determine the transfer function θ/δ_e.

c. Find the following:

(1) Undamped natural frequency (ω_n).

(2) Damping Ratio (ζ).

(3) Damped Frequency (ω_d).

8.9. For the longitudinal modes of motion, short period and phugoid, list the variables of interest.

8.10. From the T-38 wind tunnel data (Attchment 1), The following longitudinal equations of motion were derived:

$$1.565\ \dot{u} + .00452\ u + .0605\ \theta - .042\ \alpha = 0$$

$$.236\ u - 3.15\ \theta + 3.13\ \dot{\alpha} + 5.026\ \alpha = 0$$

$$.0489\ \theta + .039\ \theta + .16\ \alpha = .13\ \delta_e$$

FIND:

a. Laplace Transform of the equations.

b. 4th order characteristic equation.

8.11. The 4th order characteristic equation of Problem 8.10 was factored into the following 2nd order equations:

$$\Delta(s) = (s^2 + .00214s + .00208)\ (s^2 + 2.408s + 4.595) = 0$$

FIND:

a. Roots of the characteristic equation for short period and phugoid modes.

b. Phugoid: ω_n, ζ, ω_d, τ, $t_{1/2}$, period –

c. Short period: ω_n, ζ, ω_d, τ, $\tau_{1/2}$, period –

8.12 The short period mode of a fighter aircraft was flight tested at 30,000 ft and .8 Mach with 13,500 lb of fuel. Results were

$$\omega_{n_{SP}} = 4 \text{ radians/sec}$$

$$\zeta_{SP} = .2$$

a. The short period is to be flight tested at 30,000 ft and .8 Mach with 500 lb of fuel. (Fuel in this aircraft is distributed lengthwise along the fuselage, and the CG location for 500 lb of fuel is the same as it was for 13,500 lb of fuel.) Discuss why and how you predict ζ_{SP} and ωn_{SP} will will change at the new fuel weight.

b. The short period is to be flight tested at 5000 ft and .8 Mach with 13,500 lb of fuel. Predict the changes, if any, expected in ζ_{SP} and $\omega_{n_{SP}}$ at this low altitude point.

c. Predict the changes, if any, expected in $\omega_{n_{SP}}$ when the aircraft accelerates to 1.2 Mach ($C_{m_\alpha} \uparrow$ due to rearward movement of the aerodynamic center.)

8.13. The undamped period of the T-38 short period mode is found to be 1.9 seconds at .9 Mach at 30,000 ft. What would you predict the period to be at 50,000 ft at .9 Mach?

8.14. During a cruise performance test of the X-75B DYN, the following performance parameters were recorded:

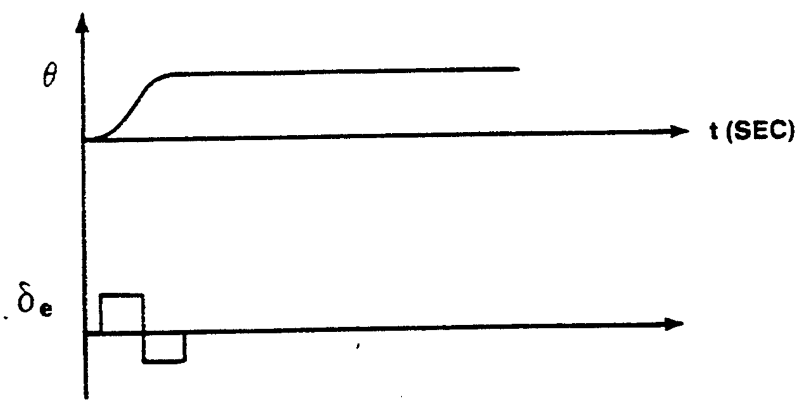
Altitude	=	20,000 ft PA
Airspeed	=	360 KTAS
ENG RPM	=	62%
Thrust	=	1,200 lb
Gross Weight	=	12,000 lb

a. Estimate the phugoid damping ratio for the given flight conditions.

b. Estimate the phugoid frequency (damped).

c. Moving the wing from full aft to full forward position should cause the short period frequency to increase/decrease and cause the damping ratio to increase/decrease?

8.15. Given the following time history traces:



What is the probable static margin, value of C_{m_α}, and static stability?

8.16. The approximation equations for the phugoid mode of motion for an aircraft are:

$$\dot{u} + .04u + 40\theta = -6\delta_e(t)$$

$$.001u - \theta = -2\delta_e(t)$$

a. Determine the characteristic equation.

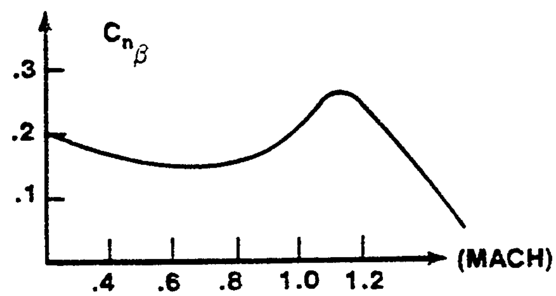
b. Calculate ζ and ω_n.

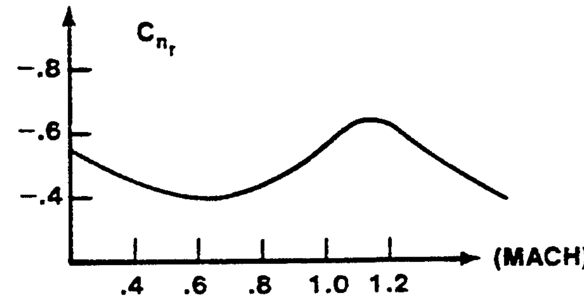
c. Is this a dynamically stable or unstable mode?

d. Calculate the time to half amplitude ($t_{1/2}$) or double amplitude (t_2).

INTENTIONALLY LEFT BLANK

8.21. Given the following wind tunnel data for the X-75B DYN:





a. Does the Dutch roll damping <u>increase/decrease</u> as Mach increases from .6 to 1.2?

b. Does the directional stability <u>increase/decrease</u> when decelerating from Mach 1.2 to .6?

8.22. You are flying a KC-135 and you transfer fuel from the outboard to the inboard wing tanks.

a. Dutch roll damping will <u>increase/decrease</u>.

b. Roll mode time constant will <u>increase/decrease</u>.

8.23. What is the stability derivative that determines the ϕ/β ratio in Dutch Roll if C_{n_β} is unchanged?

8.24. The approximation equation for an aircraft's roll mode is:

$$\dot{p} + .25p = 5.5\,\delta_a\,(t)$$

a. Determine the steady state roll rate for a step aileron input of 10°.

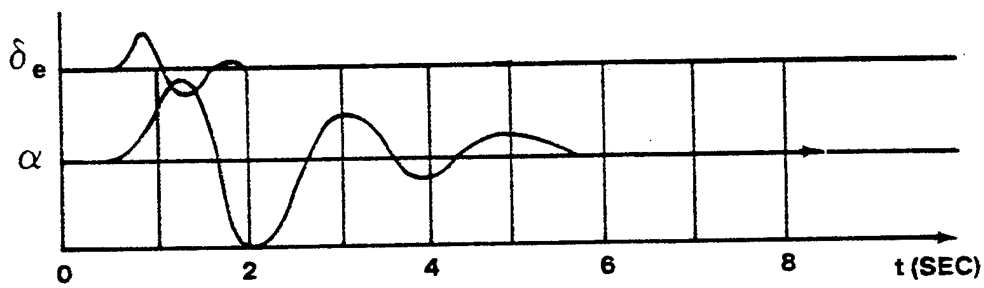
b. Determine the magnitude of roll rate after an elapsed time of:

(1) t = 1 time constant (1τ)

(2) t = 5 time constants (5τ)

8.25. For the longitudinal and lateral directional modes of motion, list the approximate period for the periodic modes and the method(s) of exciting the five modes of motion.

8.26. Given the following time history trace:



a. Type of dynamic mode represented?

b. Is this a stable dynamic mode?

c. Estimate using flight test relationships:

(1) Damping Ratio (ζ)

(2) Period of Oscillation (T)

(3) Frequency of Oscillation (ω)

(4) Undamped Natural Frequency (ω_n)

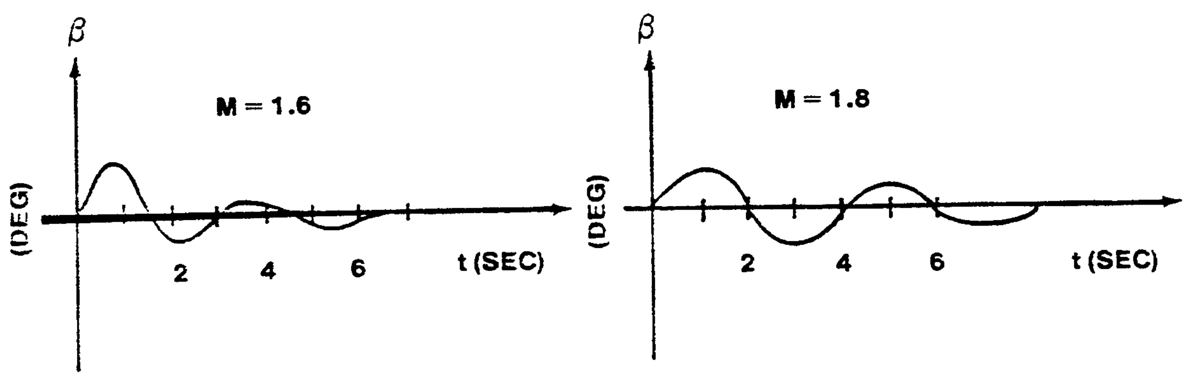
(5) Time Constant (τ)

8.27. You are a consulting engineer with the job of monitoring flight test data and giving GO/NO-GO advice on a real time basis. Today's mission is to investigate the Dutch roll mode at Mach of 1.6, 1.8, and 2.0.

Predicted values of ζ_{DR} and $\omega_{n_{DR}}$

Mach	ω_n (rad/sec)	ζ_{DR}
1.6	3.29	.30
1.8	2.23	.35
2.0	No data available	

The sideslip angle free response flight data plots for the first two test points are:



a. For both M = 1.6 and M = 1.8, estimate ζ_{DR} and f_d (cycles per second).

b. Based on the estimated values, calculate $\omega_{n_{DR}}$ for Mach = 1.6 and 1.8.

c. In light of the results observed from M = 1.6 and 1.8 points, should the M = 2.0 point be flown today? Backup your recommendation.

ATTACHMENT 1
BACKGROUND INFORMATION
FOR T-38

Flight Conditions

Altitude = 20,000 ft MSL   Density = .001267 slugs/ft^3

Mach = .8   True Airspeed = 830 ft/sec

Aircraft Dimensions:

Wing Area = 170 ft^2 Wing Span = 25.25 ft   MAC = 7.73 ft

I_y = 28,166 slug-ft^2 I_x = 1,479 slug-ft^2 I_z = 29,047 slug-ft^2

Stability Derivatives
(Wind Tunnel)
(Dimensions - per/rad)

$C_{m_\alpha} = -.16$

$C_{m_q} = -8.4$

$C_{m_{\delta_e}} = .13$

$C_{z_\alpha} = -C_{L_\alpha} = -5.026$

$C_{x_\alpha} = .084$

$C_{D\alpha} = .036$

$C_{z_q} = -C_{L_q} = -4.25$

$C_{T_u} = 1.09 \times 10^{-5}$ sec/ft

$C_L = .12$ $C_D = .0040$

$C_{y_\beta} = -1.25$

$C_{y_p} = -.10$

$C_{y_r} = .92$

$C_{l_\beta} = .0745$

$C_{l_p} = -.35$

$C_{l_r} = .055$

$C_{n_\beta} = .28$

$C_{n_r} = -.54$

$C_{n_p} = .08$

8.6 14.0 deg/sec

8.7 $\tau = 0.6$ sec

8.8 (a) $s^2 + .803s + 1.325$

(b) $\frac{\theta(s)}{\delta e(s)} = -\frac{1.38\ (s + 0.311)}{s(s^2 + 0.803s + 1.325)}$

(c) 1. $\omega_n = 1.15$ rad/sec

2. $\zeta = .35$

3. $\omega = \omega_d = 1.08$ rad/sec

8.10 (b) $s^4 + 2.408\ s^3 + 4.555\ s^2 + .015\ s + .095 = 0$

8.11 (a) $s_{1,2} = -1.204 \pm 1.77j$ (short period)

$s_{3,4} = -.0011 \pm .05j$ (phugoid)

(b) $\omega_n = .05$ rad/sec $\quad \zeta = .02, \quad \omega_d = .045$ rad/sec

$\tau = 934.6$ sec $\quad t_{1/2} = 644.9$ sec $\quad T = 137.8$ sec

(c) $\omega_n = 2.14$ rad/sec $\quad \zeta = .56, \quad \omega_d = 1.77$ rad/sec

$\tau = .83$ sec $\quad t_{1/2} = .57$ sec $\quad T = 3.5$ sec

8.12 (a) ζ_{sp} increases, $\quad \omega_{n_{sp}}$ increases

(b) ζ_{sp} increases, $\quad \omega_{n_{sp}}$ increases

(c) $\omega_{n_{sp}}$ increases

8.13 T_{50K} = 2.98 sec

8.14 (a) ζ = .07 (b) ω_d = .074 rad/sec

(c) ω_n decreases ζ increases

8.15 zero , zero , neutral static stability

8.16 (a) $s^2 + .04s + .04$

(b) ζ = .10 ω_n = .2 rad/sec

(c) stable

(d) $t_{1/2}$ = 34.5 sec

8.17 ω_n constant ζ increases

8.19 (a) $s = -.0096$ (spiral)

$s = -6.82$ (roll) $s = -.43 \pm 4.27j$ (Dutch roll)

(b) ω_n = 4.29 rad/sec ζ = .1 ω_d = 4.26 rad/sec,

$t_{1/2}$ = 1.58 sec T = 1.47 sec

(c) τ = .15 sec

(d) τ = 104.7 sec

8.20 I_x decrease C_{l_p} change slightly τ decrease

8.21 (a) increase (b) decrease

8.22 (a) increase (b) decrease

8.23 C_{l_β}

8.24 (a) p_{ss} = 220 deg/sec

(b) 1. p = 138.6 deg/sec

2. p = 218.5 deg/sec

8.26 (a) short period

(b) stable

(c) 1. ζ = .3

2. T = 2 sec

3. $\omega = \omega_d$ = 3.14 rad/sec

4. ω_n = 3.29 rad/sec

5. τ = 1.01 sec

8.27 (a) M = 1.6 ζ = .3 f_d = .33 cycles/sec

M = 1.8 ζ = .4 f_d = .25 cycles/sec

(b) M = 1.6 ω_n = 3.29 rad/sec

M = 1.8 ω_n = 1.71 rad/sec

(c) Yes, be careful of large sideslip angle excursions

BIBLIOGRAPHY

8.1 Harper, Robert P., and Cooper, George E. A Revised Pilot Rating Scale for the Evaluation of Handling Qualities. CAL Report No. 153, Cornell Aeronautical Laboratory, Inc., 1966.

8.2 Blakelock, John H. Automatic Control of Aircraft and Missiles. New York: John Wiley and Sons, 1965.

8.3 Newell, F.D. Ground Simulator Evaluations of Coupled Roll Spiral Mode Effects on Aircraft Handling Qualities. AFFDL-TR-65-39, Air Force Dynamics Laboratory, Wright-Patterson AFB, Ohio, March 1965.

8.4 Anon., Military Specification. Flying Qualities of Piloted Airplanes. MIL-F-8785C, 5 Nov 80, UNCLASSIFIED.